THE MOTION PARADOX

The 2,500-Year-Old Puzzle Behind All the Mysteries of Time and Space

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Preamble to the Paradoxes of Motion

My father was the first person to tell me about paradoxes of time. He had never heard of Zeno's paradoxes, those peculiar arguments on motion that contradict common sense and that have been misunderstood these last two and a half millennia, but was a gentleman philosopher with instinctive wisdom about the world and how it turned. My brother had just received a brand-new Schwinn bicycle with chrome fenders, a speedometer, and battery-operated horn for his birthday. Boy, was that neat. The gentleman philosopher knew just what I was thinking. To soothe my jealousy, he took me aside and told me that I was half my brother's age, but in eight years I would be three-quarters his age and that from then on there would hardly be a difference. Of course, I had no idea what he meant by three-quarters, let alone three-quarters of someone's age. When I asked how old would I have to be to catch up completely, he laughed and said that that would never happen, but that the difference would always be getting smaller. Years later, I thought I understood; but, now, rapidly gaining on my brother as I pass sixteen-seventeenths of his age, I'm just beginning to. Incidentally, my brother's bicycle was stolen shortly before his next birthday.

More than 2,000 years before my father eased my bike envy with his thought experiment, Zeno had invented similar paradoxes. Zeno argued with flawless logic that, contrary to what everyone experiences every day, nothing moves.

Zeno's four paradoxes listed in Aristotle's *Physics* are:

> The Dichotomy—*That a moving object will never reach any given point, because however near it may be, it must always first accomplish a halfway stage, and then the halfway stage of what is left and so on, and this series has no end. Therefore, the object can never reach the end of any given distance.*
>
> The Achilles—*That the swiftest racer can never overtake the slowest, if the slowest is given any start at all; because the slowest will have passed beyond his starting-point when the swiftest reaches it, and beyond the point he has then reached when the swiftest reaches it and so on. . . .*
>
> The Flying Arrow—*That it is impossible for a thing to be moving during a period of time, because it is impossible for it to be moving at an indivisible instant.*
>
> The Stadium—*That half a given period of time is equal to the whole of it; because equal motions must occupy equal times, and yet the time occupied in passing the same number of equal objects varies according as the objects are moving or stationary. The fallacy lies in the assumption that a moving body passes moving and stationary objects with equal velocity.*

The flying-arrow paradox concludes that motion is impossible. Zeno pictures an arrow in flight and considers it frozen at a single point in time. He argues that the arrow must be stationary at that instant, and that if it is stationary at that instant then it is stationary at any—and every—instant. Therefore, it does not move at all. This single paradox may bewilder, but the four together release a commotion of absurdities, profoundly questioning our models of reality.

Zeno's paradoxes raise a fundamental question about the universe: Are time and space continuous like an unbroken line, or do they come in discrete units, like a string of beads? It's a question that even today's physicists, who are reputed to be closer than ever to a theory of everything, are struggling with.

Zeno's arguments seem absurd. We know the arrow flies through the air, yet we may have some difficulty in explaining why or how we know. One may argue that the whole notion of fixing a point in time is absurd and that it makes no sense to say that an arrow appears stationary at any point in time. In mathematics, time is a variable that can be fixed by simply declaring it to be some number. We have formulas that tell us where the arrow is at any time t, so if we let t equal some specific time, then we should know the exact spot where the arrow is at that time. Yet this means that our mathematical models of motion, space, and time are merely intellectual constructions built for the convenience of easy calculations, not for the greater purpose of representing the structure of reality.

As we came to understand motion through math with greater sophistication, we shed light on Zeno's paradoxes. But only by solving the ultimate mysteries of time and space can we definitively solve the puzzles that Zeno put forth at the very dawn of science. He was ahead of his time.

HISTORY WAS NOT always generous to Zeno's inventions. At times during the past 2,000 years, his paradoxes were considered nothing more than picky sophisms of logic with little merit for continued discussion. At other times they were considered embarrassments to mathematicians' investigations of infinity and the continuum; our historians tell us that those paradoxes contributed to the Greek abandonment of such investigations.

Almost all of what we know about Zeno's life is speculation, composed from fragments and historical sources written almost a thousand years after his death. We know that he wrote a magnificent book on philosophy that was used as a textbook at Plato's Academy, but not even the smallest fragment of it has survived. The fifth-century philosopher and mathematician Proclus, our principal source of information about the early history of Greek geometry, tells us that Zeno wrote a book containing forty paradoxes, but that it was stolen before it could be published. The four known paradoxes come to us by way of Aristotle alone. Dozens of major works written by renowned scholars from Plato to Bertrand Russell have pondered the paradoxes. This literature contains a plethora of magnificently arching connections across history.

The absence of Zeno's writings warrants suspicion over whether or not the man actually existed beyond merely being a character in Plato's *Parmenides*. Despite that absence, a great deal of extant material tells of his profound philosophical ideas, and one can gather enough from them to assemble a coherent story. Plato and Diogenes Laertius provide the corners to the jigsaw puzzle of Zeno's life, Aristotle and Proclus give the edges of his philosophy, and then we fill in the rest with supposition.

After the death of Archimedes in 212 BCE, the topic of motion was effectively abandoned; it did not resurface for another 1,400 years, when Gerard of Brussels revived the mathematical works of Euclid and Archimedes and came very close to defining speed as a ratio of distance to time. A hundred years later, four Merton College mathematicians sharing ideas on the mechanics of motion were able to work out the first formulas linking acceleration to distance for a freely falling object. It has been claimed that the same math used by the Merton mathematicians solves the Achilles paradox. I'll show that while this may seem to be the case on the surface, the math in question—basic algebra—does nothing to address the underlying phenomenological problem that the paradox drives at.

Three hundred years after the Merton mathematicians, Galileo began to experiment with physical objects to measure their movement, initiating a shift toward an empirical approach to science that is still with us today. It is through Galileo that the connection between math and the physical world became solidified. Newton, Leibniz, and other mathematicians took this approach further and invented the mathematical field we now know as calculus in order to model motion.

Newton had the inspired idea that acceleration, the rate of change in velocity, was completely determined by two entities that have no apparent connection to motion—force and mass. It seemed to many that, at last, motion had been fully explained.

Math had triumphed in the explanation of the physical world. It seemed that calculus could explain the dichotomy paradox. But again, the math is merely a tool. The underlying reality that the paradox addresses is evaded.

Before the eighteenth century, time was crudely measured. Galileo used his own pulse as a measure. Today, our atomic clocks can measure a time interval as small as one-millionth of a second. (Though we have a word for one-billionth of a second—*nanosecond*—we still have no way of accurately measuring it.) But no matter how finely calibrated our clocks are, they are always measuring something discrete—an interval, a repeating signal, a duration between events. This is the heart of the problem: We measure time as a duration and think of motion as continuous. The best definition of motion we have is intricately tangled between the discrete and continuous impressions of time and space. Despite contributions by Aristotle, Galileo, Newton, and many others, for over 2,000 years nobody offered better clues about motion's deeper nature than Zeno.

The twentieth century brought relativity and quantum mechanics. Space and time were no longer thought of as separate aspects of reality; they were united into a single four-dimensional continuum. Time dilation, inconstancy of mass, and special relativity suggest that motion is indeed illusory. Motion changes mass—or is it the other way around? Quantum theory suggests that some motion is not continuous. Electrons cannot just sit anywhere within an atom. They are strictly confined to moving between discrete energy levels around an atom's nucleus. Yet we still have a hard time imagining them discretely jumping around, disrespecting our sense of continuous motion. One can't help imagining Zeno rejoicing as his paradoxes return, no longer cast off as answered by simple calculus arguments.

One thing is sure: Everything in this universe, every atom, - every molecule, is in some form of motion, whether it be simple locomotive displacement from one place to another, random molecular bombardments, or complex, astonishingly fast, unavoidable vibrations of energy transfer. And our understanding of that motion remains fundamentally paradoxical. How we have pursued the mystery of motion, and all the technological and scientific advances that pursuit has enabled, is one of the greatest stories of our civilization.

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Zeno's Visit to Athens

Athena was the gray-eyed goddess of war, fertility, art, and wisdom. Her birthday was one of those rare days when women and freed slaves were permitted to appear leisurely in public places. Imagine being in sight of the majestic Acropolis near the northwest corner of the great Athens market and gathering place. Looking southeast along the Panathenaic Way, the dusty path partly shaded by poplars and wild, hardy carob trees, you would see preparations for the Great Panathenaea festival. You would see athletes rubbed with olive oil competing for prizes in foot races, boxing, long jump, javelin throwing, and chariot racing; musicians competing with voice, kithara, and flute; and blind bards reciting Homer's epics. On this day in 450 BCE it was four years since the last great festival, just one year after the signing of a five-year truce between Athens and the other regional power, the city-state of Sparta.

Northwest, past the marketplace, through the sacred gate of the city wall and to the right lay the Ceramicus, a public square and war cemetery in the potter's district. Pentelic marble stones were being stored for the anticipated construction of the Theseion, a temple to honor Hephaestus, the skilled fire-god of the anvil with huge bulk, thin legs, "sturdy neck and hairy chest." It was quieter there, away from the loud hawking butchers, bakers, apiarists, olive pressers, wine merchants, and ironmongers lining the crushed limestone avenue leading to the festival high on the hill. Wild thyme grew through the limestone cracks near fruit vendors selling pears and figs. As Homer noted in *The Odyssey*, there, such fruit "comes at all seasons of the year and there is never a time when the West Winds' breath is not assisting, here the bud, and here the ripening fruit: so that pear after pear, apple after apple, cluster on cluster of grapes, and fig upon fig are always coming to perfection."

According to Plato, Antiphon the Sophist heard the story of Zeno's visit to Athens from his friend Pythodorus so many times that he could repeat it by heart. Parmenides, founder of the celebrated Eleatic school of philosophy, was sitting on a stone, a distinguished man in his sixties with bone-white hair. Sitting next to him was Pythodorus, a younger bearded philosopher looking particularly alert. Next to him was Aristoteles, a sunbronzed man in his thirties, lost in contemplation, and young Socrates, not yet twenty. A nearby donkey was obstinately complaining about a load of barley on its back.

Zeno of Elea, a "tall and attractive" intellectual revolutionary, was reading from his famous book on philosophy. He had come to Athens from Crotona in southern Italy with his teacher and lover Parmenides to visit Pythadorus in the Ceramicus just outside the city wall and to attend the great festival. His lines of reasoning were terribly confusing; they seemed to rely on language tricks aimed toward the mystifying suggestion that there is only one single thing in this world—the thing he called Being—and that all else is mere appearance. He argued that if a thing can be divided, its divided parts can also be divided and such divisions can continue indefinitely. From this he concluded that change, and hence motion, is not possible. He finished reading, but his audience was confused. Even Socrates was confused. He called out to Zeno.

"Zeno, what do you mean? 'If things are many,' you say 'they must be both like and unlike. But that is impossible; unlike things cannot be like, nor like things unlike.' That is what you say, isn't it?"

"Yes," replied Zeno.

The rest of his audience was as bemused as Socrates, who said, ". . . your exposition . . . seem[s] to be rather over the heads of outsiders like ourselves." Zeno was suggesting connections between the problem of plurality, being, continuity, and motion.

We have heard it all before. "And God made the firmament, and divided the waters which were under the firmament from the waters which were above the firmament: and it was so. And God called the firmament Heaven." In the book of Genesis, from the waters came two distinct things—heaven and earth. Creation is division to mark opposites—light and darkness, day and night, summer and winter, land and sea, fish and fowl, even and odd, good and evil.

What Zeno said makes sense. If two things exist, a third must exist to separate them, otherwise there would not be two things, only one. If three things exist, a fourth and fifth must exist to separate the three. To distinguish between A and B there must be a separator C, and to distinguish between A and C there must be another separator D and so on, thus proving that there must be either only one thing in this world or an infinite collection of things. "So," Socrates continued, "are you giving just one more proof that two things do not exist? Is that what you mean, or am I understanding you wrongly?"

"No," answered Zeno, "you have quite rightly understood the purpose of the whole treatise."

Zeno went on to argue that nothing changes because change would require a *becoming* and an *end to being*. "Therefore," Parmenides said, "the one which is not, not possessing being in any sense, neither ceases to be nor comes to be." He and Zeno were thinking that something in an act of change must perform that act *in time*. So change is equivalent to motion; like the arrow that can never leave the bow, change is impossible.

Zeno's arguments for motion may also be applied to the ripening of a pear. The neurologist Oliver Sacks once wrote, "I would come down to the garden in the morning and find the hollyhocks a little higher, the roses more entwined around their trellis, but, however patient I was, I could never catch them moving." We have all seen a garden of flowers, but have we ever seen the flowers growing? Like the hollyhocks, we can never catch a pear ripening, and though it may change in color, taste, texture, and even shape, it remains a pear. How does the pear get from unripe to ripe if every instant we look at it, it is in a fixed state somewhere between two extremes? Zeno's paradoxes are not only about locomotion but also more generally about change in quality and quantity.

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Zeno was a citizen of Elea, a poor Greek colony in what is now southern Italy, when Greek colonies were spreading in all directions to the banks of the Mediterranean like driftwood. Elea was "possessed of no other importance than the knowledge of how to raise virtuous citizens." Long before Alexander the Great conquered regions as far west as Marseille and as far east as India, Greece had established colonies from Carthage in North Africa to Nazareth in Palestine. In a few active centuries a small number of Greeks had developed an enormous intellectual culture connecting politics, the arts, and philosophy. They created a system of government in which a state's affairs were not simply the private interests of the king or governor, but the collective interests of its people, an experiment in democracy. Music, politics, and art combined to inspire Sophocles, Aeschylus, and Euripides to write plays of humor, tragedy, and philosophy for crowds as large as 17,000 in the Athenian outdoor theater. The Greeks discovered the mysteries of number's nature, which led them to the beginnings of what we, today, call mathematics.

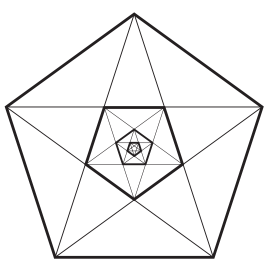
Pythagoras of Samos, who lived from about 560 to 480 BCE, was probably the most famous and charismatic mathematician of the time. We know very little about him, but that he traveled widely in the Greek world and settled in Crotona on the southeastern end of the Italian peninsula. His mathematics had a mystical aspect that drew a group of devoted students, a sect of disciples, a brotherhood that lasted for a century after his death. The Pythagoreans influenced many, including Zeno. In particular, the notion that lines were made from strings of points like threads of miniscule beads beguiled him. However, Zeno and Parmenides refuted that Pythagorean notion, and argued that if a line were made of a finite number of points, then time, too, must be built from a finite number of instants and the days would pass not in a smooth continuous flow but in discrete increments, each like a grain of sand falling in an hourglass. This was a time when growing educated classes were strongly aware of Pythagorean discoveries and their ramifications for science and geometry.

The Pythagorean brotherhood's discovery of the connection between the sizes of the sides of a right triangle blurred number theory's bond with geometry and, at the same time, gave one of the first inconsistencies of a mathematical modeling of the physical world. The Pythagorean theorem states that the sum of the squares of the lengths of the sides of a right triangle equals the square of the length of the hypotenuse. This beautiful little theorem eventually caused enormous philosophical problems for the Pythagorean brotherhood, which believed that number represented all things in this world. Legend has it that the Pythagoreans sacrificed an ox on their discovery of the famous theorem (though it's hardly likely that a strictly vegetarian cult with a belief in soul transmigration would do so).

Pythagoreans believed that everything in the world could be represented by finite arrangements of whole numbers. The number 2 represented opinion, 3 signified harmony, and 4 stood for justice. Odd numbers were male, even numbers female. So the number 5 symbolized marriage, because it was the sum of the first even number with the first odd number. The number 10 was holy because it was the sum of the generators of special dimension, $1 + 2 + 3 + 4 = 10$. The number 1 establishes a reference point, 2 points determine a unique line, 3 points not on a line determine a unique plane, and 4 points determine a tetrahedron in space. All numbers were either whole (1, 2, 3, etc.) or rational (fractions of whole numbers).

We are probably missing a lot about Pythagoras, since a covenant bound the Pythagoreans to secrecy over their master's teachings and anything else taught or discovered by the brotherhood, and moreover, the history of Greek civilization before Plato's time is murky. One of their secrets was the construction of the regular pentagram, the five-pointed star and symbol of the brotherhood that comes from connecting the corners of a pentagon. This *cosmic figure*, as the Greek historian Proclus later called it, is not easy to construct if the only tools permitted are a straight edge and compass, or, in other words, straight lines and circles. An isosceles triangle, with one angle equal to four-thirds one of the others, must be constructed. Such a triangle would have an angle of 72 degrees, and that is exactly what is needed to complete the pentagon (because a pentagon has five sides and the sum of all the angles of construction of the regular pentagram is 360 degrees).

Imagine the power these people felt upon discovering how to construct the pentagon, the five-sided figure that leads to an infinite nest of shrinking replicas of itself and an infinite expansion of growing replicas, along with its powerful numeric and geometric qualities.



The ratio of a side to a diagonal of such a pentagon gives rise to the golden mean, a number that continues to have spiritual significance among aficionados attempting to discover its hold over nature. These were also folks who believed that gods in human form watched over the actions of individuals, families, and states. From the beginning of the sixteenth century, the golden mean, whose name was not to be coined until the nineteenth century, has been considered a divine proportion because of its ubiquitous presence in the natural world and also because of how it connects simple finite constructions with infinity.

Numerical patterns also suggested to Pythagoreans that numbers were the clues to understanding the nature of the physical world. They saw numbers in music when they discovered that a plucked string produces the same note (one octave higher) as a string twice its length, and extended music theory to a harmony of the soul. They saw numbers in nature, observing the fine structures of flowers. They saw numbers in the construction of their temples, where form followed what they considered to be the spiritual beauty of divine number relationships. They saw numbers in sculpture and art as their artists sought to represent the general makeup of shared attributes, rather than the soul of an individual. They saw numbers in their plays, built on structured themes of crimes and curses. All this logic, structure, and clarity, all this love of symmetry, form, and perfection was applied to reasoning and a belief that the universe is ordered and explainable.

Math was in its youth. The invention of negative numbers would have to wait almost another 800 years for Diophantus to first mention them in his book *Arithmetica* after he found $x = -4$

to be the *absurd* solution to the equation $4x + 20 = 4$. Such absurd solutions would have to wait another 500 years before the Indian mathematician, Mahavira, actually used them and gave them a noble place in number theory. Zero had not been discovered, and neither had tomatoes, tobacco, or coffee (wine was the drink of choice, though goat's milk was tolerated).

The discovery of the Pythagorean theorem inevitably led to the discovery of incommensurables. What if you have a square with sides of length 1? The size of the diagonal would be the square root of 2. But the square root of 2 cannot be written as a ratio of two whole numbers. It is not 7/5, nor 10/7, although they are rough approximations of the square root of 2. No whole number can be divided by another to give the square root of 2. For people who worshiped number, this was extremely unnerving. Anyone discovering relationships such as $1^2 + 3 = 2^2$, $2^2 + 5 = 3^2$, $3^2 + 7 = 4^2$, etc., might conceive mystical notions of the powers of pattern and credit them to some deity's impressive wisdom of order. Essentially, one ruler cannot measure both the side and the diagonal of a square. These early Greeks had discovered an immeasurable part of space. Zeno surely knew about this discovery when he posed his paradoxes questioning the continuity of space and time.

Later, in the early part of the twentieth century, Bertrand Russell wrote, "The problem first raised by the discovery of incommensurables proved, as time went on, to be one of the most severe and at the same time most far-reaching problems that have confronted the human intellect in its endeavor to understand the world."

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PHILOSOPHIZING GREEKS OF the fifth century BCE continued a 200-year attempt that began with Thales of Miletus to articulate a more scientific system of knowledge, to reject any supernatural explanations of nature, and to question the essence of things. Rational criticism and debate replaced speculative thought and established myth. Thales believed that the earth rocking on water caused thunder. His attempt to explain the nature of thunder might be called primitive because it rested on false hypotheses, but modern because it dodged the popular belief in the supernatural.

A theory of the atom, albeit crude, was suggested by the Pythagoreans and developed by Anaxagoras, author of a book reputed to be a complete account of the natural world (now, sadly, lost). The argument that complex things must be made of simpler things was further advanced by Empedocles, a rich doctor from the island that is present-day Sicily. He saw those irreducible things as earth, fire, air, and water, but was careful to point out that each of these elements stood for a wide variety of substances. Water, for example, was a term applied to liquids such as molten metals as well as drinkable fluids. Air would have meant any gas, including those expelled from cattle in the fields. All this makes almost modern sense if one views the classification of matter as solid, liquid, gas, and heat.

Heat? Is that matter? Fire seems to be more of an action. Fire can be used to change the three states of matter or combinations of them into the things we see, or to change one state to another—ice to water, water to steam. Empedocles, in his wisdom, listed three material things together with a device for combining, shaping, and altering those material things. Without fire, the world of things must rely on accidental collisions and linkages to change. With it, the local smithy can learn the art of Hephaestus to hammer the world into new shapes and things from the elements. Empedocles says it this way:

> Just as painters, when they decorate offerings—
> men well taught by skill in their art—
> take the many-colored pigments in their hands,
> and, harmoniously mixing them, some more some less,
> make from them shapes resembling all things,
> creating trees and men and women
> and beasts and birds and fish that live in the sea
> and even gods, long-lived and highest in honor:
> so let not deceit persuade your mind that there is any
>   other source
> for the countless mortal things we see.
> But know this clearly, having heard the tale from a god.

Again, that begs the question of what we get when we take a very close look at the elements and see them, even if we have to rely on imagination, as indivisible things of incredibly small size. Atomism holds that all things consist of substances so small they escape our senses. These indivisible *atoms* are thought to be of many forms, shapes, and sizes, becoming perceptible only after massive collections of them entangle, hook, and bind together through motion and collisions in the void. What they become depends on their shape, arrangement, and position. These groupings of atoms can make the imperceptible perceptible, but they can also untangle and unhook to make the visible invisible. There is an astounding resemblance between this atomic theory and our own twenty-first-century one, where we believe that all matter is composed of atoms and that we only see the matter when enough atoms are compounded to make a substance visible. We see gold when there are enough gold atoms to make the collection of gold atoms visible.

Fifth-century BCE atomic science was imaginative opinion, supported by dialogues against equally creative alternative theories. There were no measurements of atomic weight, nor were there instruments to examine matter any finer than what could be seen with the best pair of eyes, but there were consequences that led to further questioning.

Leucippus, the fifth-century BCE Greek philosopher whose thinking was very much influenced by Zeno and Parmenides, was the founder of the first atomic theory of matter, asserting that atoms consist of imperceptibly minute and indivisible particles that differ only in shape and position. This wonderful theory, which was developed later by his pupil Democritus and led to unexpected results in science, bears directly on the Pythagorean trouble with measuring the diagonal of a square. It is likely that the Pythagoreans thought of a line as a string of atoms, so a line twice as long would contain twice as many atoms. Given that, there must be a definite ratio between any two lengths, because the number of atoms on each line must be finite and hence the ratio of lengths must be a fraction whose numerator is the number of atoms contained in one line and whose denominator is the number of atoms contained in the other.

The atomist argument is that there is a difference between the physical atom and the geometrical point. The atom is indivisible and indestructible, whereas the point is an imagined notion with no physical substance. They reasoned that material substances could be divided as finely as humanly possible, and from there, imagined a moment when no further division would be possible. "Take a wooden stick," they reasoned. "Cut it in two parts. And every day cut the longer piece in half. Continue this cutting day by day, indefinitely. One day it will be hard to claim that the longer end is still a stick, yet easy to maintain that it is still a piece of wood. But how many days will pass before the wood becomes non-wood?" Even the smallest speck of sawdust is still wood.

Anaxagoras knew the group of men who gathered in the Ceramicus in Athens to listen to Zeno, and he was a good friend of Euripides and Pericles. He wrote a book on physics, his only book, offering a complete account of the natural world, arguing that there is a bit of everything in everything. How does a human hair grow from nothing? The answer, Anaxagoras would suggest, is that the food digested by the human already contains hair and everything else within it, imperceptible to our senses. According to Anaxagoras, wood, even in the form of minute particles of sawdust, contains a bit of every other substance, including human hair, a notion stemming from the philosophy of Empedocles and Heraclitus of Ephesus declaring that materials might be changed but not destroyed. These men might have wondered how oil disappears from an oil lamp burning through the night, but, had they imagined the answer, they would have foretold our modern conservation laws, which say that energy is not lost; it can only be converted into other forms of energy or matter.

Empedocles had the reasonably correct idea that everything could be derived from four elements, for we should "hear first the four roots of all things: bright Zeus, life-bringing Hera, Aidoneus, and Nestis, who waters with her tears the mortal fountains." More directly, he says:

> Come and I will tell you . . .
> from which all the things we now see come to be:
> earth and the billowy sea and the damp air
> and the Titan ether, binding everything in a circle.

If the elementary substances of the universe are only earth, air, fire, and water, then how is it that other substances appear to be different from those four? Once again, we are told not to trust appearances. Should we trust our senses or rely on our ability to reason? The problem of divisibility is central to the problem of trusting the senses. Heraclitus, nicknamed "The Riddler," felt that everything is subject to change and was the first philosopher to profess a distinction between mind and sense.

"It's one thing for the eyes and ears to witness sound and sight," he would say, "but what good are they, if the mind cannot interpret what they hear and see?"

Do we obtain knowledge of nature through reason alone, or do we acquire it through sense alone?

Parmenides felt that we only perceive change through reason. For him, one is persuaded by the virtues of experience, intuition, and compelling forces suggesting that things could not be otherwise. He was referring to this kind of persuasion in his poem, *The Way of Truth.*

> The only ways of enquiry that can be thought of:
> the one way, that it *is* and cannot *not-be*,
> is the path of Persuasion, for it attends upon Truth.

For him, knowledge of nature was based exclusively on reason, which in his time was a newly defined activity, and not, as the Pythagorean experimentalists had supposed, based on observation. "Engrained habit and experience may tempt the use of the blind eye, echoing ear and tongue as instruments of knowledge, but let reason be the test," he would say. "Beware of the senses."

Heraclitus, too, was occupied with the question of which of the two, observation or reason, was the way of truth. For him it was observation, "Because," he would say, "everything changes. So how could reason, which must be fixed, lead to truth about a world where everything changes from one moment to the next?" Not a bad argument, but Parmenides would attack it and ask, "Then how does Earth change to Water or Water to Vapor? Water is less dense than Earth and Vapor less dense than Water. To change from one to the other empty space must be introduced. But empty space is nothingness, which does not exist. Hence there is no such thing as change. The world is one spherically solid motionless universe, incapable of change by the argument that nothingness cannot be something."

Reason had become a new game, complete with that wonderful new logical principle, *contradiction*—after all, a thing cannot be and not be at the same time, just as nothingness cannot be the thing that makes vapor from water. It was a game that would spur intellectual thought over hundreds and now thousands of years to the heights of scientific knowledge.

zeno argued that movement is impossible because in order for a body to move any distance it must first get to half the distance, then half the remaining distance, and so on, forever reaching half of some remaining distance—hence, never reaching the full distance. Aristotle wrote that this paradox suggests that movement is "impossible because, however near the mobile is to any given point, it will always have to cover the half, and then the half of that, and so on without limit before it gets there." Zeno wrote all this in a book, which he claimed was stolen, and which is reported to have contained "forty different paradoxes following from the assumption of plurality and motion." How devastating his loss must have been, writing day after day on scrolls of papyrus, planning ahead, and anticipating each new thought before cutting the skin and sewing in new patches.

There are many variations on this argument, and surely Zeno had considered them. It means any task can never be finished, for in order for it to finish, half the task must be done, and when that is accomplished, half the remaining task must be finished, and so on ad infinitum. The task is general: anything from reading this book to winning gold (a hundred amphorae of olive oil) in a Great Panathenaea chariot race. Mathematicians may simply deny the paradox by claiming that the sum $1/2 + 1/4 + 1/8 + \ldots$ is equal to 1, but they cannot answer the question of *how* the task is actually completed in reality. Mathematics tells us that it happens without explaining why.

At some point after reading his treatise in Athens, Zeno left the Ceramicus with Parmenides, Pythodorus, Aristoteles, and Socrates to retreat to the home of Pythodorus. They walked through a courtyard, through stables, up a few steps to a porch, then through the women's quarters and into a long room with cushioned seats around the walls facing a central hearth over a stone floor. It is likely they encountered preparations for a symposium that would happen late in the evening after the Panathenaea festival—oil lamps being filled, as well as large urns for wine and water.

Here Zeno argued that if one shot an arrow at a target, then examined it at any fixed instant of time, the arrow would appear stationary. If it is stationary at any instant, how can it be in motion? How can it ever even leave the bow, let alone move through the air and reach its target?

One may argue that the very notion of fixing a point in time is absurd and that it makes no sense to say "an arrow appears stationary at any point in time." But in mathematics, time is a variable that can be fixed by declaring it to be some number of units of time from some starting time. Mathematical formulas tell us where an arrow is at any time t, so if we let t equal some specific time, say two seconds after leaving the bow, we should know the exact spot where the arrow is when $t = 2$. But is there any such thing as exactly two seconds, or even an exact spot? We know that if we really try to take a picture of the arrow when $t = 2$, we must have the shutter open for an entire interval of time surrounding $t = 2$. The shutter cannot open and close at the same instant.

Mathematical representations of physics are models that are constructed in the mind. The key to understanding Zeno's arguments is to understand the connection between what it means, both mathematically and physically, to let the time variable be equal to a constant. The mathematician is the conjuror here. Stop time to see the arrow stationary? Yes, that would, indeed, seem to disturb movement, but what we see is not the real arrow; it is another arrow moving in the mind.

Continuity suggests an *uninterrupted* path. We move from here to there without passing through gaps in space. To us, motion seems uninterrupted. Yet, we envision objects moving through space on a line or curve made from an aggregate of points representing numbers, perhaps the distance from one end of the curve. For any number on a number line there is no such thing as a *next* number. So, how do we move from one point to the next, if there is no such thing as a next point? This is the salient arrow in Zeno's quiver. If a path is an aggregate of points, then an object's motion cannot generate a path.

Tobias Dantzig, the twentieth-century author of several popular books on mathematics, put it beautifully: "When we see a ball in flight we perceive the motion as a whole and not as a succession of infinitesimal jumps. But neither is a mathematical line the true, or even the fair, representation of a wire. Man has for so long been trained in using these fictions that he has come to prefer the substitute to the genuine article."

And that's just it. We have been trained in using fictions. We see a ball in flight and presume that what we see is what actually happens. But the mind, not the eye, is the seeing organ. Consider the zoetrope, that nineteenth-century parlor-room toy, in which no more than a dozen still images of a man in various anatomical positions give the illusion that the man is running.

The films we watch are more advanced illusions of continuity. A one-hour film is composed of 86,400 individual still images, yet we see the scenes pass by with utter smoothness. The seventy-two still images on film of a ball in flight for three seconds may look just the same as the real ball in flight. Doubling the number of still images and doubling the speed of the film may not give the viewer any more realistic sense of continuity. There is something biologically magical in that threshold number of frames per second (twenty-four) that tricks the mind into thinking that what we are seeing is continuous. But the mind seems to be able to process far more than twenty-four frames per second, integrating information faster than a film can deliver.

Perhaps there is a good motive for Zeno's motion arguments. Perhaps physical motion simply cannot be represented by mathematical space and time under arbitrarily small intervals beyond measurable experience. The great nineteenth-century mathematicians David Hilbert and Paul Bernays put forward a disturbing answer:

> Actually there is also a much more radical solution of the paradox. This consists in the consideration that we are by no means obliged to believe that the mathematical space-time representation of motion is physically significant for arbitrarily small space and time intervals; but rather have every basis to suppose that that mathematical model extrapolates the facts of a certain realm of experience, namely the motions within the orders of magnitude hitherto accessible to our observation. . . .

Zeno was known as "the two-tongued Zeno" because he often argued both sides of his own arguments, which usually involved either the infinite or the infinitesimal. Two of his paradoxes assume that space and time consist of a finite number of points and instants, while two others make the opposite assumption. There are only three ways out of these paradoxes: either we agree that (1) space and time consists of points and instants, and there are an infinite number of points within any interval; (2) that there are no points and instants in space; or (3) we deny the real existence of space and time altogether.

He was asking such questions more than two millennia before any thoughts of quantum mechanics and relativity, already posing questions contrasting our experiences of motion and our sense of continuity with logical explanations of what we assume to be *reality.* We seem to be comfortable with motion at the macroscopic level by intuiting what we expect to happen through experience, but with no sensory experience at the microscopic level we run into trouble and counterintuitive wonders.

Anyone who believes the atomist argument that all matter consists of atoms and that the atom is indivisible and indestructible must also believe that a moving object must pass from one spot to the next as time passes from one instant to the next. Of course, Zeno was assuming that time moves from past to future through a sequence of successive instants. He was also assuming something far more acceptable: If the object is always moving forward, it cannot be in the same place at two distinct instants of time. We know that Zeno's followers were confused by the meaning of his paradox, but more than twenty-four centuries have passed for intelligent people to have made some sense of it. Even Aristotle seemed to have been confused when he mentioned it in his *Physics.* We now have a clearer understanding of what Zeno could have meant.

Consider three adjacent points labeled *A, B,* and *C.* By this I mean that *B* is immediately to the right of *A* and that *C* is immediately to the right of *B.* In one indivisible instant, an object cannot travel from point *A* to point *C.* If it could, there would be no instant when it could be at point *B.* Of course, this is absurd, because that would mean all motion must take place at the same speed. The only way out of this is to reject the thought that points or instants are consecutive, i.e., arranged in a hierarchy from left to right and vice versa. This leads to equally puzzling thoughts about how a moving body gets from one point to another. If an object moved from *A* to *C,* there must have been a moment when it was at a point *B* between *A* and *C.* And there must have been a moment when it was at a point between *A* and *B.* This can go on indefinitely.

The stadium paradox asks us to imagine three lines, each either above or below another. Mark the points. The top line has points labeled A_1, A_2, A_3, etc.; the middle line has points labeled B_1, B_2, B_3, etc.; and the bottom line has points labeled C_1, C_2, C_3, etc. The letter indicates the position of the line and the number indicates the position of the point on the line. Now imagine that the lines line up so that the numbers are each above or below each other.

$$\begin{array}{ccc} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{array}$$

Next, imagine that the top line is stationary, the middle line is moving to the left at a constant speed *s,* and the lower line is moving to the right at the same speed *s.*

$$\begin{array}{ccccc} & A_1 & A_2 & A_3 & \\ B_1 & B_2 & B_3 & & \\ & & C_1 & C_2 & C_3 \end{array}$$

Suppose that the line is made up of discrete points. You may have noticed that before these lines moved, A_2, B_2, and C_2 lined up as a column of points, but on the very first instant of movement, the points B_3 and C_1 line up under A_2. It seems that B_3 skipped over C_2 to line up with C_1. In other words, there was never an instant when A_2, B_3 and C_2 lined up as a column. What happened? The answer strikes at Zeno's point. We made one fallacious assumption: that the line is made up of discrete points. We could view Zeno's stadium argument as an indirect proof that the line is not made of discrete points.

Though nature is fantasized as continuous—both by our brains, such as when we are watching a film, and by reason, as argued in Zeno's stadium paradox—she does make jumps. The piece of wood that is divided often enough seems to stay wood for many divisions, but at some point, there will be a specific division when the wood dust suddenly becomes something other than wood. This is the first of several jumps as we continue to split our pieces of matter down to the atom. Eventually, we are left with splitting operations that can take place only in the mind.

3

The World Through Aristotle's Eyes

In 343 BCE, Aristotle would take long walks from palace at Pella to a little gate by the Axius River in Macedonia. He was born in Stagira, a large town near the three fingers of Macedonia jutting into the Aegean, where wild fig trees struggled to grow in rocky soil. Those trees rarely bore fruit, though occasionally someone could find and pluck a lonely fig hidden in their foliage. Aristotle loved to walk, and would often stroll the dusty sandstone road alongside the city wall from the palace to the gate. He was nicknamed "The Peripatetic." Though he was wrong about many details, his gift to the world of knowledge—a contribution that guided the West for more than a thousand years—was an explanation of almost everything.

By the end of his life he had written 337 books on topics ranging from love to medicine. Yet his attire revealed a man calling attention to himself; his clothes were conspicuously fanciful, as were his carefully trimmed hair and the rings on his fingers. His face was clean-shaven; his body garlanded with ornaments and jewels. He tutored Alexander the Great in botany, zoology, and physics. Alexander was only thirteen, and not yet emperor of Macedonia, Greece, North Africa, Persia, and the Punjab of India.

Aristotle had a broad concept of *nature*, one that was very different from the concept we have today. For him, the study of nature was the study of "all things that move or change, or that come and go either in some sense of passing from 'here' to 'there,' or in the more extended sense of passing from 'this' to 'that,' which latter phrase is equivalent to 'becoming something that it was not'—a solid becoming a liquid or a hot thing becoming cold."

The field of change is broad enough to include things that fall, rise, sink, or expand, and even souls that might transmigrate. A stone rolls down a hillside, cold becomes hot, a bubble is born in boiling water, a block of Pentelic marble becomes the bust of Hermes, a mind is persuaded by a convincing argument, or—to paraphrase Aristotle—an uncultivated man becomes cultivated. These all involve motion in its broadest sense.

Aristotle's theses imply that the cultivation of intelligence leads to the joys of life. He believed that daily experience and sensations demand the development of an understanding of material nature—hopeful and inspiring stuff, after the bleak Platonic opinion that all knowledge falls short of unattainable ideals. It may be difficult to imagine a time when all science was simply thinking about nature, without tests or experiments, a time when the man on the street could hypothesize about the universe by feeling that something is true and making a good argument for its case, a time when there were no laboratories or statistical samplings to measure probabilities. In the fourth century BCE, reasoning was all that was needed to make a scientific case. Aristotle built his cases from first principles—that is, from indisputable statements—claiming that reasoning is not possible without first principles, definitions, and hypotheses. If he should want to talk of change, he would start by hypothesizing that "wherever anything changes, it always changes either from one thing to another, or from one magnitude to another, or from one quality to another, or from one place to another; but there is nothing that embraces all these kinds of change in common, and is itself neither substantive nor quantitive nor qualitive nor pertaining to any of the other categories. . . ."

Motion for him meant more than just *locomotion*—the movement of an object from one place to another. It meant movement in quality (black to white), or in form (the ripening pear), or quantity (growth in size), or displacement (locomotion). Nature to him was the cause of all things that move, change, or pass from *this* to *that.* "Nature is the principle of movement and change," he wrote. "And since we are interested in Nature, we must understand what 'movement' is. First, we should understand that movement is 'continuous' and that continuity implies the concept of the 'illimitable.' " It was an amazing revelation.

In his book *On Movement*, Aristotle claimed that in order to have movement at all, we must first have continuity, and in order to have continuity we must have division without limit. He was not thinking of division of physical objects such as a stick, which can be divided only up to the point of its atomic indivisibles, but of the space and time in which the stick sits.

Anyone reading *On Movement* might ask why something that moves must move through divisible time and space, and the answer is reminiscent of Zeno's: Anything that changes must change in time and space, and hence time must be divisible, for nothing that cannot be divided in time can be made to move in space. Aristotle argued a thing that is undergoing change cannot change from *here* to *there* or from *this* to *that* all at once, for if it did there would have to be an instant when the whole thing became *this* from *that*. He was trying to connect *time* to *change* by making the argument that time is continuous and, therefore, change must be, too.

Aristotle argued for the connection between mathematical continuity and real-world continuity by observing that a traveling object cannot skip positions—it must move from one position to the next. But he was not an atomist. For him, the continuity of space did not imply the infinite division of the object traveling through space. This seems contradictory, and is reminiscent of Zeno's arguments. How can an object move from one position to the next without space coming in discrete units?

Aristotle wrote, "Movement cannot occur except in relation to place, void and time." He also wrote, "These four things—place, void, movement and time—are universal conditions common to all natural phenomena."

Movement can only happen by direct touch between a moving agent and the moving thing—the stone carver's chisel whittles the stone, the potter's hands shape the clay, and the weaver rapidly pushes the weft and shuttle back and forth across a warp through a perfectly synchronized opening and closing heald. For the case of the moving stick, the front pulls the rear or the rear pushes the front.

Aristotle said, “Taking the initiator of movement to mean not that for the sake of which the movement takes place but that which sets it going, we may say that the initiator must be in direct touch with the thing it immediately moves; and by this I mean that there can be nothing between them. This is true of every mover and the moved it directly acts upon.”

Hearing involves air particles hitting the eardrum. Seeing involves light waves stimulating the retina. Aristotle could not have known about rods and cones on the retina, and yet, they are in accord with his concept of nature. What about emotions—fear, anger, love? Aristotle attributed those to blood flow. He claimed anger to be “the seething of the blood, or heat in the region of the heart.” For him, mind was in the heart, and the eyes were windows to the soul. And all things could be explained by one thing touching and moving another.

Direct contact between the mover and the moved applies to all kinds of motion—locomotion from one place to another, whether the moved is being moved by itself or not; qualitative motion, as in a ripening pear; or quantitative motion, as in the growth or shrinkage of a herd of goats. But anything that moves must move from somewhere to someplace else, or from one state of being to another in some span of time.

But just as motion needs time, time needs motion. In his *Physics*, Aristotle wrote, “So, just as there would be no time if there were no distinction between this ‘now’ and that ‘now,’ but it was always the same ‘now’; in the same way there appears to be no time between two 'nows' when we fail to distinguish between them." Time and motion are therefore different but inseparable. He asks us to try to imagine time without movement or movement without time. It's impossible. "Even if it were dark and we were conscious of no bodily sensations, but something were 'going on' in our minds, we should, from that very experience, recognize the passage of time." For Aristotle, motion is a gateway into understanding the very fabric of the universe.

TIME IS THE measure of motion—and vice versa. Today we measure time in terms of physical locomotion. Time is simply a recording that separates physical "befores" and "afters." Every moderately precise clock—from Galileo's swinging pendulum to our modern atomic clocks (which oscillate at billions of cycles per second)—measures time by some form of stop-and-go mechanism.

Aristotle presents us with a brainteaser. If all motion were to cease in the universe for an interval of time, what could we possibly mean by that interval? If motion is not taking place, then the time span of the interval is not either; the interval collapses as though there never was one. In other words, every time interval must represent the motion of something in the universe.

There is also a hint of relativity in Aristotle's conception of time. We may ask, What would happen if only one thing in the universe were in motion? We would have to answer that the interval would exist and have some particular measurement, based on the motion of the single moving object. But what would happen to the measure of time when a second object begins to move? Aristotle's answer is that if one object covers less distance in the same time interval than another, then it must be moving "slower" and that time is still the conceptual measure; that is, "we do not speak of time itself as 'swift or slow,' but as consisting of 'many or few' of the units in which it is counted, or as 'long and short' when we regard the continuum . . . for abstract numbers are in no case swift or slow, though the counting of them may be." In effect, he is measuring speeds qualitatively and following a Greek tradition of explaining phenomena through the use of proportions and analogies. Yet we do speak of "swift or slow" as relative terms when we consider distance covered as "great or small" in the time interval considered.

Aristotle believed that if time is continuous, then so is space. Yet time is divided by this curious thing we know as "now"; and, by the same reasoning, so is space. The position of any object in motion is marked and divided by its "now" place in space. But that does not exclude the concept of a smallest unit of time or space. Aristotle surely understood that an interval could be infinitely divided, but his conception of infinity grants that we can always imagine a "beyond"—a potential for continuing indefinitely; that our minds have the power to continue to divide a line or an interval of time as often as we like. But those divisions refer only to rational numbers, the only measurements Aristotle would have known about.

Aristotle uses this potential infinity to argue that Zeno's dichotomy paradox—the argument that a moving object must repeatedly pass a succession of halfway points before getting to its end position—is based on the false belief that it is impossible for a thing to take up an infinite number of positions in a finite amount of time. In effect, a moving object would have to "count" infinitely many numbers before the end of its journey.

Modern mathematics has models that make it possible to perform an infinite number of tasks in a finite amount of time by playing the dichotomy paradox in reverse. David Hilbert's famous infinite hotel trick is a good example: Somewhere in math wonderland there is a hotel with an infinity of rooms numbered 1, 2, 3, and so on.The hotel is always full, but there is always room for one more guest. The manager moves the occupants of room 1 to room 2, the occupants of room 2 to room 3, and so forth. This frees up room 1 for the new arrival. This may seem impossible to accomplish in a finite amount of time, given that the occupants must move in real space and real time. But if the occupant in the first room takes 1/2 hour to move, the occupant in room 2 takes 1/4 hour, and the occupant in the n-th room takes $1/2^n$ hour, then the infinity of moves will be finished in just one hour.

However, Aristotle claims that Zeno had made false assumptions in asserting that it is impossible for a thing to take up an infinite number of positions in a finite amount of time. He points out that time and space are *equally* divisible without limit and therefore there should be not be any surprise that a person can pass through an infinite number of positions in an infinite collection of instants. But there is more to his refutation of Zeno's dichotomy paradox. He claims that when the path of motion is bisected, the motion is interrupted; the bisected point is considered twice—once at the end of the first segment and again at the beginning of next segment.

Modern topology—the branch of mathematics concerned with special properties that are independent of distance measurement—would be disturbed about this, for it would assume that the point of division lies in one segment or the other, but not in both. So here is Aristotle's argument. If time is continuous and the points of time are represented as points of space, then the point's position must be represented by both the past and future. He argues that Zeno is presuming that if a white object were changing to not-white in a period of time divided into two intervals—*A*, during which it is white, and *B*, during which it is non-white—then there must be some instant *C* when it is both white and non-white; in other words, we are left with the devilishly perplexing contradiction that *C* belongs to both *A* and *B*.

Aristotle argues that the contradiction is based on something he doesn't believe is true: the Pythagorean notion that time is a string of atomic moments, one following directly from its neighbor with nothing in between. This awareness of the nature of number density is significant—it was not fully appreciated by mathematicians before the seventeenth century and the invention of calculus, which depends on the density of irrational numbers in the set of real numbers.

Aristotle argues that if something is moving at one instant it must have already been moving, though perhaps slower or faster. If space and time are both continuous, without "next" points or atomic moments, and if time is merely an intangible numerical scale in our consciousness representing motion, then time is a continuous measure of change in position. It follows that there is no change in position in any instant of time, but it does not follow that no change is taking place.

His definition of "being at rest" means that from one instant to another entirely different instant, the body in question and all its parts occupy the same place. Moreover, he asserts that time is indefinitely divisible. Therefore, when Zeno claims that his flying arrow "does not move" at an indivisible instant, Aristotle agrees that it and all its parts occupy the same place at that instant, but that does not mean it is at rest, for, in order to be at rest, it and all its parts must occupy the same place for a period of time. In other words, whatever is in motion changes position as time continuously moves on; it does not matter what is happening in a single instant.

However, Zeno anticipated his refuters and cleverly designed his four paradoxes to trap them between assumptions of divisibility and indivisibility of time and space. The first two (the dichotomy and Achilles) assume that space and time are infinitely divisible while the second two (the arrow and the stadium) make the opposite assumption.

To refute the Achilles paradox, Aristotle reduces it to the dichotomy by correctly noting that it too is a kind of division of space, not by halves (as the dichotomy supposes), but by a ratio of the speeds of the racers. He also correctly notes that Zeno dupes us into focusing on the moments before Achilles overtakes the tortoise by designing the argument as a catching-up question. Yes, Achilles does not overtake the tortoise while the tortoise is ahead, but we tend to forget that the race continues to the finish line, which may or may not be beyond the point where Achilles overtakes the tortoise.

The fourth paradox seems to be the real trap. In effect, a corollary is that all speeds are equal, for if time and space are made from indivisible atomic instants and points respectively, then a body is forced to pass one atom of space in one atom of time. If that were not the case, then the body would have to pass one atom of space in more (or less) than one atom of time, which would make the atom of time divisible. But Aristotle seems to have misunderstood the point. His brief criticism simply attacks the hypothesis when he says, "The fallacy lies in his assuming that a moving object takes an equal time in passing another object equal in dimensions to itself, whether that other object is stationary or in motion; which assumption is false."

All these arguments seemed to center on the possibility of motion and whether or not time and space were continuous. Cause was a different question. And Aristotle argued that all motion is caused by an external agent, but avoids the question of how that agent continues to do its thing when not in contact with the thing being moved. "If a thing is in motion it is, of necessity, being kept in motion by something." What is that something? His answer is that it is either something within the moving object that keeps it moving, or some other moving agent in contact with it. In his view, motion must be started by something that is already moving and that motion continues only by contact with something that continues to push or pull. The image here is an infinite succession of agents each being pushed or pulled by its neighbor. The idea that a body in motion will continue in motion unless acted upon would have inverted his understanding of cause. He had no concept of inertia the way we do to explain why a stone continues to travel after it leaves the hand that throws it. That concept was still a millennium away.