

Multiverses and physical cosmology

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Abstract

The idea of a multiverse – an ensemble of universes – has received increasing attention in cosmology, both as the outcome of the originating process that generated our own universe, and as an explanation for why our universe appears to be fine-tuned for life and consciousness. Here we carefully consider how multiverses should be defined, stressing the distinction between the collection of all possible universes, and ensembles of really existing universes that are essential for an anthropic argument. We show that such realised multiverses are by no means unique. A proper measure on the space of all really existing universes or universe domains is needed, so that probabilities can be calculated, and major problems arise in terms of realised infinities. As an illustration we examine these issues in the case of the set of Friedmann-Lemaître-Robertson-Walker (FLRW) universes. Then we briefly summarise scenarios like chaotic inflation, which suggest how ensembles of universe domains may be generated, and point out that the regularities which must underlie any systematic description of truly disjoint multiverses must imply some kind of common generating mechanism. Finally, we discuss the issue of testability, which underlies the question of whether multiverse proposals are really scientific propositions.

1 Introduction

The idea of a multiverse has been proposed as the only scientifically based way of avoiding the fine-tuning required to set up the conditions for our seemingly very unlikely universe to exist. Stephen Weinberg (2000), for example, uses it to explain the value of the cosmological constant, which he relates to anthropic issues. Martin Rees (2001) employs it to explain the whole set of anthropic coincidences (Barrow and Tipler 1986), that is, to explain why our universe is a congenial home for life. These and similar proposals have been triggered by the dawning awareness among many researchers that there may be many other existing universes besides ours. This possibility has received strong stimulation from proposals like Andrei Linde's (1983,

1990) chaotic inflationary scenario, in which the origin of our own observable universe region naturally involves the origin of many other similar expanding universe regions.

There is however a vagueness about the proposed nature of multiverses. They might occur in various ways, discussed by Weinberg (2000) and Tegmark (2003). They might originate naturally in different times and places through meta-cosmic processes like chaotic inflation, or in accord with Lee Smolin’s (1999) cosmic Darwinian vision. In the latter case, an ensemble of expanding universe regions grow from each other following gravitational collapse and re-expansion, where natural selection of universes through optimisation of black hole production leads to bio-friendly universe regions. This is an intriguing idea, but with many uncertain steps – in particular no proof has been given of the last step, that the physics that maximises black hole production also favours life. They might be associated with the multi-universe Everett-Wheeler-type interpretation of quantum mechanics. Or perhaps multiverses can be truly disjoint collections of universes (see Sciama 1993, Rees 2001, Tegmark 1998, 2003).

Some refer to the separate expanding universe regions in chaotic inflation as ‘universes’, even though they have a common causal origin and are all part of the same single spacetime. In our view (as ‘uni’ means ‘one’) *the Universe* is by definition the one unique connected¹ existing spacetime of which our observed expanding cosmological domain is a part. We will refer to situations such as in chaotic inflation as a *Multi-Domain Universe*, as opposed to a completely causally disconnected *Multiverse*. Throughout this paper, when our discussion pertains equally well to disjoint collections of universes (multiverses in the strict sense) and to the different domains of a Multi-Domain Universe, we shall for simplicity simply use the word “*ensemble*”. When an ensemble of universes are all sub-regions of a larger connected spacetime - the “universe as a whole” - we have the multi-domain situation, which should be described as such. Then we could reserve “multiverse” for the collection of genuinely disconnected “universes” – those which are not locally causally related.

So far, none of these ideas have been developed to the point of actually describing such ensembles of universes in detail, nor has it been demonstrated that a generic well-defined ensemble will admit life. Some writers tend to imply that there is only one possible multiverse (characterised by “all that can exist does exist”). This vague prescription actually allows a vast variety of different realisations with differing properties, leading to major problems in the definition of the ensembles and in averaging, due to the lack of a well-defined measure and the infinite character of the ensemble itself. Furthermore it is not at all clear that we shall ever be able to accurately delineate the class of all possible universes.

The aim of this paper is to help clarify what is involved in a full description of an ensemble of universes. Our first contribution is clarifying what is required in order to describe the space of possible universes, where much hinges on what we regard as ‘possible’. However that is only part of what is needed. It is crucial to recognise that anthropic arguments for *existence* based on ensembles of universes with specific properties require an *actually existing ensemble* with all the required properties. For purposes of providing an explanation of existence, simply having a conceptually possible ensemble is not adequate - one needs a link to objects or things that actually exist, or to mechanisms that make them exist.

¹“Connected” implies “Locally causally connected”, that is all universe domains are connected by C^0 timelike lines which allow any number of reversals in their direction of time, as in Feynman’s approach to electrodynamics. Thus it is a union of regions that are causally connected to each other, and transcends particle and event horizons; for examples all points in de Sitter space time are connected to each other by such lines.

The second contribution of this paper is to show how an actually existing ensemble may be described in terms of a space of possible universes, by defining a distribution function (discrete or continuous) on the space of possible universes. This characterizes which of the theoretically possible universes have been actualised in the ensemble - it identifies those that have actually come into existence. This leads us to our third point: the problems arising when it is claimed that there is an actually existing ensemble containing an infinite number of universes or of expanding universe regimes. Actually existing infinities are very problematic.

There are fundamental issues that arise in considering ensembles of actually existing universes: what would explain the existence of an ensemble, and its specific properties? Why should there be this particular ensemble, rather than some other one? Why should there be any regularity at all in its properties? The fourth point we make is that if all the universes in an ensemble show regularities of structure, then that implies some common generating mechanism. Some such structuring is necessary if we are to be able to describe a multiverse with specified properties - a coherent description is only possible through the existence of such regularities. Hence a multiverse consisting of completely causally disconnected universes is a problematic concept.

The issue of testability is a further important consideration: Is there any conceivable direct or indirect way of testing for existence of an ensemble to which our universe belongs? Our fifth point is that there is no way we can test any mechanism proposed to impose such regularities: they will of necessity always remain speculative. The sixth point is to argue that existence of multiverses or ensembles is in principle untestable by any direct observations, and the same applies to any hypothesized properties we may suppose for them. However certain observations would be able to disprove existence of some multi-domain ensembles. It is only in that sense that the idea is a testable proposition.

It is clear that in dealing with multiverses one inevitably runs up against philosophical and metaphysical issues, for example concerning the ability to make scientific conclusions in the absence of observational evidence, and in pursuing the issue of realised infinities. A companion more philosophically oriented paper will pursue those issues.

2 Describing Ensembles: Possibility

To characterise an ensemble of existing universes, we first need to develop adequate methods for describing the class of all possible universes. This requires us to specify, at least in principle, all the ways in which universes can be different from one another, in terms of their physics, chemistry, biology, etc.

2.1 The Set of Possible Universes

The basis for describing ensembles or multiverses is contained in the structure and the dynamics of a space \mathcal{M} of all possible universes m , each of which can be described in terms of a set of states s in a state space \mathcal{S} . Each universe in \mathcal{M} will be characterised by a set \mathcal{P} of distinguishing parameters p , which are coordinates on \mathcal{S} . Some will be logical parameters, some will be numerical constants, and some will be functions or tensor fields defined in local coordinate neighbourhoods for s . Each universe m will evolve from its initial state to some final state according to the dynamics operative, with some or all of its parameters varying as it does so. The course of this evolution of states

will be represented by a path in the state space \mathcal{S} , depending on the parametrisation of \mathcal{S} . Thus, each such path (in degenerate cases a point) is a representation of one of the universes m in \mathcal{M} . The coordinates in \mathcal{S} will be directly related to the parameters specifying members of \mathcal{M} . The parameter space \mathcal{P} has dimension N which is the dimension of the space of models \mathcal{M} ; the space of states \mathcal{S} has $N + 1$ dimensions, the extra dimension indicating the change of each model's states with time, characterised by an extra parameter, e.g., the Hubble parameter H which does not distinguish between models but rather determines what is the state of dynamical evolution of each model. Note that N may be infinite, and indeed will be so unless we consider only geometrically highly restricted sets of universes.

It is possible that with some parameter choices the same physical universe m will be multiply represented by this description; thus a significant issue is the equivalence problem – identifying which different representations might in fact represent the same universe model. In self-similar cases we get a single point in \mathcal{S} described in terms of the chosen parameters \mathcal{P} : the state remains unchanged in terms of the chosen variables. But we can always get such variables for any evolution, as they are just comoving variables, not necessarily indicating anything interesting is happening dynamically. The interesting issue is if this invariance is true in physically defined variables, e.g., expansion normalised variables, then physical self-similarity is occurring.

The very description of this space \mathcal{M} of possibilities is based on an assumed set of laws of behaviour, either laws of physics or meta-laws that determine the laws of physics, which all universes m have in common; without this, we have no basis for setting up its description. The detailed characterisation of this space, and its relationship to \mathcal{S} , will depend on the matter description used and its behaviour. The overall characterisation of \mathcal{M} therefore must incorporate a description both of the geometry of the allowed universes and of the physics of matter. Thus the set of parameters \mathcal{P} will include both geometric and physical parameters.

The space \mathcal{M} has a number of important subsets, for example:

1. $\mathcal{M}_{\text{FLRW}}$ – the subset of all possible exactly Friedmann-Lemaître-Robertson-Walker (FLRW) universes, described by the state space $\mathcal{S}_{\text{FLRW}}$ (in the case of dust plus non-interacting radiation a careful description of this phase space has been given by Ehlers and Rindler1989).
2. $\mathcal{M}_{\text{almost-FLRW}}$ – the subset of all perturbed FLRW model universes. These need to be characterised in a gauge-invariant way (see e.g. Ellis and Bruni 1989) so that we can clearly identify those universes that are almost-FLRW and those that are not.
3. $\mathcal{M}_{\text{anthropic}}$ – the subset of all possible universes in which life emerges at some stage in their evolution. This subset intersects $\mathcal{M}_{\text{almost-FLRW}}$, and may even be a subset of $\mathcal{M}_{\text{almost-FLRW}}$, but does not intersect $\mathcal{M}_{\text{FLRW}}$ (realistic models of a life-bearing universe like ours cannot be exactly FLRW, for then there is no structure).
4. $\mathcal{M}_{\text{Observational}}$ – the subset of models compatible with current astronomical observations. Precisely because we need observers to make observations, this is a subset of $\mathcal{M}_{\text{anthropic}}$.

If \mathcal{M} truly represents all possibilities, one must have a description that is wide enough to encompass *all* possibilities. It is here that major issues arise: how do we decide what all the possibilities are? What are the limits of possibility? What classifications of possibility are to be included? “All that can happen happens” must imply

all possibilities, as characterised by our description in terms of families of parameters: all allowed values must occur, and they must occur in all possible combinations. The full space \mathcal{M} must be large enough to represent all of these possibilities, along with many others we cannot even conceive of, but which can nevertheless in principle also be described by such parameters. An interesting related point has been pointed to us by Jean-Phillipe Uzan: it may be that the larger the possibility space considered, the more fine-tuned the actual universe appears to be - for with each extra possibility that is included in the possibility space, unless it can be shown to relate to already existing parameters, the actual universe and its close neighbours will live in a smaller fraction of the possibility space. For example if we assume General Relativity then there is only the parameter G to measure; but if we consider scalar-tensor theories, then we have to explain why we are so close to General Relativity now. Hence there is a tension between including all possibilities in what we consider, and giving an explanation for fine tuning.

From these considerations we have the first key issue:

Issue 1: What determines \mathcal{M} ? Where does this structure come from? What is the meta-cause that delimits this set of possibilities? Why is there a uniform structure across all universes m in \mathcal{M} ?

The meta-question is whether any of these questions can be answered scientifically. We return to that at the end.

2.2 Adequately Specifying Possible Anthropic Universes

When defining any ensemble of universes, possible or realised, we must specify all the parameters which differentiate members of the ensemble from one another at any time in their evolution. The values of these parameters may not be known or determinable initially in many cases – some of them may only be set by transitions that occur via processes like symmetry breaking within given members of the ensemble. In particular, some of the parameters whose values are important for the origination and support of life may only be fixed later in the evolution of universes in the multiverse.

We can separate our set of parameters \mathcal{P} for the space of all possible universes \mathcal{M} into different categories, beginning with the most basic or fundamental, and progressing to more contingent and more complex categories. Ideally they should all be independent of one another, but we will not be able to establish that independence for each parameter, except for the most fundamental cosmological ones. In order to categorise our parameters, we can doubly index each parameter p in \mathcal{P} as $p_j(i)$ such that those for $j = 1 - 2$ describe basic physics, for $j = 3 - 5$ describe the cosmology (given that basic physics), and $j = 6 - 7$ pertain specifically to emergence and life (we must include the latter if we seriously intend to address anthropic issues). Our characterisation is as follows:

1. $p_1(i)$ are the basic physics parameters within each universe, excluding gravity - parameters characterising the basic non-gravitational laws of physics in action, related constants such as the fine-structure constant α , and including parameters describing basic particle properties (masses, charges, spins, etc.) These should be logical parameters or dimensionless parameters, otherwise one may be describing the same physics in other units.
2. $p_2(i)$ are basic parameters describing the nature of the cosmological dynamics, e. g., $p_2(1) = 1$ indicates Einstein gravity dominates, $p_2(1) = 2$ indicates Brans-

Dicke theory dominates, $p_2(1) = 3$ indicates Electro-magnetism dominates, etc. Associated with each choice are the relevant parameter values, e.g., $p_2(2) = G$, $p_2(3) = \Lambda$, and in the Brans-Dicke case $p_2(4) = \omega$. If gravity can be derived from more fundamental physics in some unified fundamental theory, these will be related to $p_1(i)$; for example the cosmological constant may be determined from quantum field theory and basic matter parameters.

3. $p_3(i)$ are cosmological parameters characterising the nature of the matter content of a universe. These parameters encode whether radiation, baryons, dark matter, neutrinos, scalar fields, etc. occur, in each case specifying the relevant equations of state and auxiliary functions needed to determine the physical behaviour of matter (e.g. barotropic equations of state and the potential function for scalar fields). These are characterisations of physical possibilities for the macro-states of matter arising out of fundamental physics, so the possibilities here will be related to the parameters in $p_1(i)$. Realistic representations will include all the above, but simplified ensembles considered for exploratory purposes may exclude some or many of them.
4. $p_4(i)$ are physical parameters determining the relative amounts of each kind of matter present in the specific cosmological solutions envisaged, for example the density parameters Ω_i of various components at some specific stage of its evolution (which then for example determine the matter to anti-matter ratio and the entropy to baryon ratio). The matter components present will be those characterised by $p_3(i)$.
5. $p_5(i)$ are geometrical parameters characterising the spacetime geometry of the cosmological solutions envisaged- for example the scale factor $a(t)$, Hubble parameter $H(t)$, and spatial curvature parameter k in FLRW models. These will be related to $p_4(i)$ by the gravitational equations set in $p_2(i)$, for example the Einstein Field Equations.
6. $p_6(i)$ are parameters related to the functional emergence of complexity in the hierarchy of structure, for example allowing the existence of chemically complex molecules. Thus $p_6(1)$ might be the number of different types of atoms allowed (as characterised in the periodic table), $p_6(2)$ the number of different states of matter possible (solid, liquid, gas, plasma for example), and $p_6(3)$ the number of different types of molecular bonding. These are emergent properties arising out of the fundamental physics in operation, and so are related to the parameters set in $p_1(i)$.
7. $p_7(i)$ are biologically relevant parameters related specifically to the functional emergence of life and of self-consciousness, for example $p_7(1)$ might characterise the possibility of supra-molecular chemistry and $p_7(2)$ that of living cells. This builds on the complexity allowed by $p_6(i)$ and relates again to the parameter set $p_1(i)$.

It is important to note that these parameters will describe the set of possibilities we are able to characterise on the basis of our accumulated scientific experience. The limits of our understanding are relevant here, in the relation between what we conceive of as this space of possibilities, and what it really is. There may be universes which we believe are possible on the basis of what we know of physics, that may in fact not be possible. There may also be universes which we conceive of as being impossible for one reason or another, that turn out to be possible. And it is very likely that we simply may not be able to imagine or envisage all

the possibilities. However this is by no means a statement that “all that can occur” is arbitrary. On the contrary, specifying the set of possible parameters determines a uniform high-level structure that is obeyed by all universes in \mathcal{M} . We see, then, that a possibility space \mathcal{M} is the set of universes (one-parameter sets of states \mathcal{S}) obeying the dynamics characterised by a parameter space \mathcal{P} , which may be considered to be the union of all allowed parameters $p_j(i)$ for all i, j as briefly discussed above:

$$\mathcal{M} = \{\mathcal{S}, \mathcal{P}\}, \quad \mathcal{P} = \cup_{i,j} p_j(i).$$

Because the parameters \mathcal{P} determine the dynamics, the set of paths in \mathcal{S} characterising individual universes m are determined by this prescription. In some particular envisaged ensemble, some of these parameters (‘class parameters’) may be fixed across the ensemble, thus defining a class of universes considered, while others (‘member parameters’) will vary across the ensemble, defining the individual members of that class. Thus

$$\mathcal{P} = \mathcal{P}_{class} \cup \mathcal{P}_{member}.$$

As we consider more generic ensembles, class parameters will be allowed to vary and so will become member parameters. In an ensemble in which all that is possible happens, all parameters will be member parameters; however that is so hard to handle that we usually analyse sub-spaces characterised by particular class parameters.

2.3 Describing the Geometry of Possible Universes

Cosmological models are characterised by a preferred timelike vector field $u : u^a u_a = -1$, usually the fluid flow vector (Ellis 1971a), but sometimes chosen for other reasons, e.g. to fit local symmetries. To describe a cosmological spacetime locally we must give a description of its (generally inhomogeneous and anisotropic) geometry via suitable parameters $p_5(i)$. This description may be usefully given in terms of a tetrad basis as follows (see Ellis and van Elst 1999, Wainwright and Ellis 1996, Uggla, *et al.* 2003):

Feature 1: a set of local coordinates $\mathcal{X} = \{x^i\}$ must be chosen in each chart of a global atlas. This will in particular have a time coordinate t which will be used to characterise evolution of the universe; this should be chosen in as uniform as possible a way across all the universes considered, for example it may be based on surfaces of constant Hubble parameter H for the preferred vector field u .

Feature 2: in each chart, to determine the geometry we must be given the components $\mathcal{E} = [e^i_a(x^j)]$ of an orthonormal tetrad with the fluid flow vector chosen as the timelike tetrad vector ($a, b, c, ..$ are tetrad indices; four of these components can be set to zero by suitable choice of coordinates). Together the coordinates and the tetrad form the reference frame

$$\mathcal{P}_{frame} \equiv \{\mathcal{X}, \mathcal{E}\}. \quad (1)$$

The metric tensor is then

$$ds^2 = g_{ij}(x^k) dx^i dx^j = \eta_{ab} e^a_i(x^k) e^b_j(x^l) dx^i dx^j$$

where η_{ab} is the Minkowski metric:

$$\eta_{ab} = e_a \cdot e_b = \text{diag}(-1, +1, +1, +1)$$

(because the tetrad is orthonormal) and $e_j^b(x^l)$ are the inverse of $e_a^i(x^j)$:

$$e_a^i(x^j)e_i^b(x^j) = \delta_a^b.$$

Thus the metric is given by

$$ds^2 = -\left(e_i^0 dx^i\right)^2 + \left(e_i^1 dx^i\right)^2 + \left(e_i^2 dx^i\right)^2 + \left(e_i^3 dx^i\right)^2 \quad (2)$$

The basic geometric quantities used to determine the spacetime geometry are the rotation coefficients Γ_{bc}^a of this tetrad, defined by

$$\Gamma_{bc}^a = e_j^a e_{c;k}^j e_b^k.$$

They may conveniently be given in terms of geometric quantities

$$\mathcal{P}_{\text{geometry}} \equiv \{\dot{u}_\alpha, \theta, \sigma_{\alpha\beta}, \omega_{\alpha\beta}, \Omega_\gamma, a^\alpha, n_{\alpha\beta}\}. \quad (3)$$

characterised as follows:

$$\begin{aligned} \Gamma_{\alpha 00} &= \dot{u}_\alpha, \\ \Gamma_{\alpha 0\beta} &= \frac{1}{3}\theta + \sigma_{\alpha\beta} - \omega_{\alpha\beta}, \\ \Gamma_{\alpha\beta 0} &= \epsilon_{\alpha\beta\gamma}\Omega^\gamma, \\ \Gamma_{\alpha\beta\gamma} &= a_{[\alpha}\delta_{\beta]\gamma} + \epsilon_{\gamma\delta[\alpha}n_{\beta]}^\delta + \frac{1}{2}\epsilon_{\alpha\beta\delta}n_{\delta}^\gamma, \end{aligned}$$

where \dot{u}_α is the acceleration of the fluid flow congruence, θ is its expansion, $\sigma_{\alpha\beta} = \sigma_{(\alpha\beta)}$ is its shear ($\sigma_b^b = 0$), and $\omega_{\alpha\beta} = \omega_{[\alpha\beta]}$ its vorticity, while $n_{\alpha\beta} = n_{(\alpha\beta)}$ and a_α determine the spatial rotation coefficients (see Wainwright and Ellis 1996, Ellis and van Elst 1999). Greek indices (with range 1 – 3) indicate that all these quantities are orthogonal to u^a . They are spacetime fields, although in particular high-symmetry cases they may be independent of many or of all the coordinates. The Jacobi identities, Bianchi identities, and Einstein field equations can all be written out in terms of these quantities, as can the components $E_{\alpha\beta}$, $H_{\alpha\beta}$ of the Weyl tensor (see Ellis and van Elst 1999). Except in the special cases of isotropic spacetimes and locally rotationally symmetric spacetimes (see Ellis 1967, van Elst and Ellis 1996), the basis tetrad can be chosen in an invariant way so that three of these quantities vanish and all the rest are scalar invariants.

Thus the geometry is determined by the 36 spacetime functions in the combined set $(\mathcal{E}, \mathcal{P}_{\text{geometry}})$ with some chosen specification of coordinates \mathcal{X} , with the metric then determined by (2). For detailed dynamical studies it is often useful to rescale the variables in terms of the expansion (see Wainwright and Ellis 1995, Uggla et al 2003 for details). Note that the same universe may occur several times over in this space; the *equivalence problem* is determining when such multiple representations occur. We do not recommend going to a quotient space where each universe occurs only once, as for example in the dynamical studies of Fischer and Marsden (1979), for the cost of doing so is to destroy the manifold structure of the space of spacetimes. It is far better to allow multiple representations of the same universe (for example several representations of the same Bianchi I universe occur in the Kasner ring in the space of Bianchi models, see Wainwright and Ellis 1996) both to keep the manifold structure intact and because then the dynamical structure becomes clearer.

Feature 3: To determine the global structure, we need a set of composition functions relating different charts in the atlas where they overlap, thus determining the global topology of the universe.

Together these are the parameters $p_5(i)$ needed to distinguish model states. A particular model will be represented as a path through those states. The nature of that evolution will be determined by the matter present.

2.4 Describing the Physics of Possible Universes

Feature 4: To determine the matter stress-energy tensor we must specify the quantities

$$\mathcal{P}_{\text{matter}} \equiv \{\mu, q_\alpha, p, \pi_{ab}, \Phi_A\} \quad (4)$$

for all matter components present, where μ is the energy density, q_α is the momentum flux density, p is the pressure, $\pi_{ab} = \pi_{(ab)}$ the anisotropic pressure ($\pi^b_b = 0$), and Φ_A ($A = 1..A_{\text{max}}$) is some set of internal variables sufficient to make the matter dynamics deterministic when suitable equations of state are added (for example these might include the temperature, the entropy, the velocity v^i of matter relative to the reference frame, some scalar fields and their time derivatives, or a particle distribution function). These are parameters $p_4(i)$ for each kind of matter characterised by $p_3(i)$. Some of these dynamical quantities may vanish (for example, in the case of a ‘perfect fluid’, $q_\alpha = 0$, $\pi_{ab} = 0$) and some of those that do not vanish will be related to others by the equations of state (for example, in the case of a barotropic fluid, $p = p(\mu)$) and dynamic equations (for example the Klein Gordon equation for a scalar field). These equations of state can be used to reduce the number of variables in $\mathcal{P}_{\text{matter}}$; when they are not used in this way, they must be explicitly stated in a separate parameter space \mathcal{P}_{eos} in $p_3(i)$. In broad terms

$$\mathcal{P}_{\text{eos}} \equiv \{q_\alpha = q_\alpha(\mu, \Phi_A), \quad p = p(\mu, \Phi_A), \\ \pi_{ab} = \pi_{ab}(\mu, \Phi_A), \quad \dot{\Phi}_A = \dot{\Phi}_A(\Phi_A)\}. \quad (5)$$

Given this information the equations become determinate and we can obtain the dynamical evolution of the models in the state space; see for example Wainwright and Ellis (1996), Hewitt et al (2002), Horwood et al (2002) for the case of Bianchi models (characterised by all the variables defined above depending on the time only) and Uggla et al (2003), Lim et al (2003) for the generic case.

Feature 5: However more general features may vary: the gravitational constant, the cosmological constant, and so on; and even the dimensions of spacetime or the kinds of forces in operation. These are the parameters $\mathcal{P}_{\text{physics}}$ comprising $p_1(i)$ and $p_2(i)$. What complicates this issue is that some or many of these features may be emergent properties, resulting for example from broken symmetries occurring as the universe evolves. Thus they may come into being rather than being given as initial conditions that then hold for all time.

Initially one might think that considering all possible physics simply involves choices of coupling constants and perhaps letting some fundamental constant vary. But the issue is more fundamental than that. Taking seriously the concept of including *all* possibilities in the ensembles, the space of physical parameters $\mathcal{P}_{\text{physics}}$ used to describe \mathcal{M} , the parameters $p_2(i)$ might for example include a parameter $p_{\text{grav}}(i)$ such that: for $i = 1$ there is no gravity; for $i = 2$ there is Newtonian gravity; for $i = 3$ general relativity is the correct theory at all energies – there is no quantum gravity

regime; for $i = 4$ loop quantum gravity is the correct quantum gravity theory; for $i = 5$ a particular version of superstring theory or M-theory is the correct theory.

Choices such as these will arise for all the laws and parameters of physics. In some universes there will be a fundamental unification of physics expressible in a basic “theory of everything”, in others this will not be so. Some universes will be realised as branes in a higher dimensional spacetime, others will not.

2.5 The Anthropic subset

We are interested in the subset of universes that allow intelligent life to exist. That means we need a function on the set of possible universes that describes the probability that life may evolve. An adaptation of the Drake equation (Drake and Shostak) gives for the probability of intelligent life in any particular universe m in an ensemble,

$$P_{life}(m) = F * \Pi \quad (6)$$

where the existence of a habitat for life is expressed by the product

$$\Pi = P_{gal} * R * f_S * f_p * n_e \quad (7)$$

and the coming into existence of life, given such a habitat, is expressed by the product

$$F = f_l * f_i. \quad (8)$$

Here P_{gal} is the probability of galaxies forming in the universe considered, R is the average rate of star formation in galaxies, f_S is the fraction of these stars that can provide a suitable environment for life (they are ‘Sun-like’), f_p is the fraction of stars that are surrounded by planetary systems, n_e is the mean number of planets in each such system that are suitable habitats for life (they are ‘Earth-like’), f_l is the fraction of such planets on which life actually originates, and f_i represents the fraction of those planets on which there is life where self-conscious beings develop. The anthropic subset of a possibility space is that set of universes for which $P_{life}(m) > 0$.

The probabilities $\{P_{gal}, R, f_S, f_p, n_e, f_l, f_i\}$ are functions of the physical and cosmological parameters characterised above, so there will be many different representations of this parameter set depending on the degree to which we try to represent such interrelations. The astrophysical issues expressed in the product Π are the easier ones to investigate. We can in principle make a cut between those consistent with the eventual emergence of life and those incompatible with it by considering each of the factors in Π in turn, taking into account their dependence on the parameters $p_1(i)$ to $p_5(i)$, and only considering the next factor if all the previous ones are non-zero (an approach that fits in naturally with Bayesian statistics and the successive allocation of relevant priors). In this way we can assign “bio-friendly intervals” to the possibility space \mathcal{M} . If Π is non-zero we can move on to considering similarly whether F is non-zero, based on the parameters $p_6(i)$ to $p_7(i)$ determining if true complexity is possible, which in turn depend on the physics parameters $p_1(i)$ in a crucial way that is not fully understood. It will be impossible at any stage to characterise that set of the multiverse in which *all* the conditions *necessary* for the emergence of self-conscious life and its maintenance have been met, for we do not know what those conditions are (for example, we do not know if there are forms of life possible that are not based on carbon and organic chemistry). Nevertheless it is clear that life demands unique combinations of many different parameter values that must be realised simultaneously. When we look at

these combinations, they will span a very small subset of the whole parameter space (Davies 2003, Tegmark 2003).

If we wish to deal with specifically human life, we need to make the space \mathcal{M} large enough to deal with all relevant parameters for this case, where free will arises. This raises substantial extra complications, discussed in the companion (more philosophical) paper.

2.6 Parameter space revisited

It is now clear that some of the parameters discussed above are dependent on other ones, so that while we can write down a more or less complete set at varying levels of detail they will in general not be an independent set. There is a considerable challenge here: to find an independent set. *Inter alia* this involves solving both the initial value problem for general relativity and the way that galactic and planetary formation depend on fundamental physics constants (which for example determine radiation transfer properties in stars and in proto-planetary gas clouds), as well as relations there may be between the fundamental constants and the way the emergent complexity of life depends on them. We are a long way from understanding all these issues. This means we can provide necessary sets of parameter values but cannot guarantee completeness or independence.

3 The Set of Realised Universes

We have now characterised the set of possible universes. But in any given ensemble, they may not all be realised, and some may be realised many times. The purpose of this section is to set up a formalism making clear which of the *possible* universes (characterised above) occur in a specific *realised* ensemble.

3.1 A distribution function describing an ensemble of realised universes

In order to select from \mathcal{M} a set of realised universes we need to define on \mathcal{M} a distribution function $f(m)$ specifying how many times each type of possible universe m in \mathcal{M} is realised. The function $f(m)$ expresses the contingency in any actualisation – the fact that not every possible universe has to be realised, and that any actual universe does not have to be realised as a matter of necessity. Things could have been different! Thus, $f(m)$ describes the *ensemble of universes* or *multiverse* envisaged as being realised out of the set of possibilities. If these realisations were determined by the laws of necessity alone, they would simply be the set of possibilities described by \mathcal{M} . In general they include only a subset of possible universes, and multiple realisations of some of them. This is the way in which chance or contingency is realised in the ensemble².

²It has been suggested to us that in mathematics terms it does not make sense to distinguish identical copies of the same object: they should be identified with each other because they are essentially the same. But we are here dealing with physics rather than mathematics, and with real existence rather than possible existence, and then multiple copies must be allowed (for example all electrons are identical to each other; physics would be very different if there were only one electron in existence).

The class of models considered is determined by all the parameters held constant ('class parameters'). Considering the varying parameters for the class ('member parameters'), some will take only discrete values, but for each one allowed to take continuous values we need a volume element of the possibility space M characterised by parameter increments $dp_j(i)$ in all such varying parameters $p_j(i)$. The volume element will be given by a product

$$\pi = \prod_{i,j} m_{ij}(m) dp_j(i) \quad (9)$$

where the product $\prod_{i,j}$ runs over all continuously varying member parameters i, j in the possibility space, and the m_{ij} weight the contributions of the different parameter increments relative to each other. These weights depend on the parameters $p_j(i)$ characterising the universe m . The number of galaxies corresponding to the set of parameter increments $dp_j(i)$ will be dN given by

$$dN = f(m)\pi \quad (10)$$

for continuous parameters; for discrete parameters, we add in the contribution from all allowed parameter values. The total number of galaxies in the ensemble will be given by

$$N = \int f(m)\pi \quad (11)$$

(which will often diverge), where the integral ranges over all allowed values of the member parameters and we take it to include all relevant discrete summations. The probable value of any specific quality $p(m)$ defined on the set of galaxies will be given by

$$P = \int p(m)f(m)\pi \quad (12)$$

Such integrals over the space of possibilities give numbers, averages, and probabilities.

Hence, a (realised) ensemble E of galaxies is described by a possibility space \mathcal{M} , a measure π on \mathcal{M} , and a distribution function $f(m)$ on \mathcal{M} :

$$E = \{\mathcal{M}, \pi, f(m)\}. \quad (13)$$

The distribution function $f(m)$ might be discrete (e. g., there are 3 copies of universe m_1 and 4 copies of universe m_2 , with no copies of any other possible universe), or continuous (e.g. characterised by a given distribution of densities Ω_i). In many cases a distribution function will exclude many possible universes from the realisation it specifies.

Now it is conceivable that all possibilities are realised – that all universes in \mathcal{M} exist at least once. This would mean that the distribution function

$$f(m) \neq 0 \text{ for all } m \in \mathcal{M}.$$

But there are an infinite number of distribution functions which would fulfil this condition, and so a really existing 'ensemble of all possible universes' is not unique. In such ensembles, all possible values of each distinguishing parameter would be predicted to exist in different members of the multiverse in all possible combinations with all other parameters at least once, but they may occur many times. One of the problems is that this often means that the integrals associated with such distribution functions would diverge, preventing the calculation of probabilities from such models (see our treatment of the FLRW case below).

From this consideration we have the second key issue:

Issue 2: What determines $f(m)$? What is the meta-cause that delimits the set of realisations out of the set of possibilities?

The answer to this question has to be different from the answer to *Issue 1*, precisely because here we are describing the contingency of selection of a subset of possibilities from the set of all possibilities, determination of the latter being what is considered in *Issue 1*. Again, the meta-question is whether this can be answered scientifically.

3.2 Measures and Probabilities

It is clear that $f(m)$ will enable us to derive numbers and probabilities relative to the realisation it defines only if we also have determined a unique measure π on the ensemble, characterised by a specific choice of the weights $m_{ij}(m)$ in (9), where these weights will depend on the $p_j(i)$. There are three issues here.

First, what may seem a “natural” measure for \mathcal{M} in one set of coordinates will not be natural in another set of coordinates. Hence the concept of a measure is not unique, as is illustrated below in the FLRW case. This is aggravated by the fact that the parameter space will often contain completely different kinds of quantities (density parameters and the values of the gravitational constant and the cosmological constant, for example), and assigning the weights entails somehow assigning a relative weighting between these quite different kinds of quantities.

Second, it is possible that we might be able to assign probabilities $\chi(m)$ to points of \mathcal{M} from some kind of physical argument, and then predict $f(m)$ from these, following the usual line of argument for determining entropy in a gas. However, we then have to determine some reason why $\chi(m)$ is what it is and how it then leads to $f(m)$. In the entropy case, we assume equal probability in each phase space volume; why should that hold for an ensemble of universes? Realising such probabilities seems to imply a causal mechanism relating the created members of the multiverse to one another so they are not in fact causally disjoint, otherwise, there is no reason why any probability law (Gaussian normal, for example) should be obeyed. We will return to this point later.

Finally, the relevant integrals may diverge. In that case, assigning mean values or averages for physical quantities in an ensemble of universes is problematic (see Kirchner and Ellis 2003 and references therein).

3.3 The Anthropic subset

The expression (6) can be used in conjunction with the distribution function $f(m)$ of galaxies to determine the probability of life arising in the whole ensemble:

$$P_{life}(E) = \int f(m) * P_{gal} * R * f_S * f_p * n_e * f_l * f_i * \pi \quad (14)$$

(which is a particular case of (12) based on (6)). An anthropic ensemble is one for which $P_{life}(E) > 0$. If the distribution function derives from a probability function, we may combine the probability functions to get an overall anthropic probability function—for an example see Weinberg et al discussed below, where it is assumed that P_{gal} is the only relevant parameter for the existence of life. This is equivalent to assuming that

$R * f_S * f_p * n_e * f_t * f_i = 1$. This assumption might be acceptable in our physically realised universe, but there is no reason to believe it would hold generally in an ensemble because these parameters will depend on other ensemble parameters, which will vary.

3.4 Problems With Infinity

When speaking of multiverses or ensembles of universes – possible or realised – the issue of infinity often crops up. Researchers often envision an *infinite* set of universes, in which all possibilities are realised. Can there really be an infinite set of really existing universes? We suggest that, on the basis of well-known philosophical arguments, the answer is No.

There is no conceptual problem with an infinite set – countable or uncountable – of *possible* or *conceivable* universes. However, as stressed by David Hilbert (1964), it can be argued that a *really existing* infinite set is not possible. As he points out, following many others, the existence of the actually infinite inevitably leads to well-recognised unresolvable contradictions in set theory, and thus in definitions and deductive foundations of mathematics itself (Hilbert, pp. 141-142). His basic position therefore is that “Just as operations with the infinitely small were replaced by operations with the finite which yielded exactly the same results . . ., so in general must deductive methods based on the infinite be replaced by finite procedures which yield exactly the same results.” (p. 135) He concludes, “Our principle result is that the infinite is nowhere to be found in reality. It neither exists in nature nor provides a legitimate basis for rational thought . . . The role that remains for the infinite to play is solely that of an idea . . . which transcends all experience and which completes the concrete as a totality . . .” (Hilbert, p. 151). Others (see Spitzer 2000 and Stoeger 2003 and references therein) have further pointed out that realised infinite sets are not constructible – there is no procedure one can in principle implement to complete such a set – they are simply incompletable. But, if that is the case, then “infinity” cannot be arrived at, or realised. On the contrary, the concept itself implies its inability to be realised! This is precisely why a realised past infinity in time is not considered possible from this standpoint – since it involves an infinite set of completed events or moments. There is no way of constructing such a realised set, or actualising it.

Thus, it is important to recognise that infinity is not an actual number we can ever specify or reach – it is simply the code-word for “it continues without end”. Whenever infinities emerge in physics – such as in the case of singularities – we can be reasonably sure, as is usually recognised, that there has been a breakdown in our models. An achieved infinity in any physical parameter (temperature, density, spatial curvature) is almost certainly *not* a possible outcome of any physical process – simply because it means traversing in actuality an interval of values which never ends. We assume space extends forever in Euclidean geometry and in many cosmological models, but we can never prove that any realised 3-space in the real universe continues in this way – it is an untestable concept, and the real spatial geometry of the universe is almost certainly not Euclidean. Thus Euclidean space is an abstraction that is probably not realised in physical practice. In the physical universe spatial infinities can be avoided by compact spatial sections, either resultant from positive spatial curvature or from choice of compact topologies in universes that have zero or negative spatial curvature, (for example FLRW flat and open universes can have finite rather than infinite spatial sections). Future infinite time is never realised: rather the situation is that whatever time we reach, there is always more time available. Much the same applies to claims of a past infinity of time: there may be unbounded time available in the past in principle,

but in what sense can it be attained in practice? The arguments against an infinite past time are strong – it’s simply not constructible in terms of events or instants of time, besides being conceptually indefinite.³

The same problem of a realised infinity may be true in terms of the supposed ensembles of universes. It is difficult enough conceiving of an ensemble of many ‘really existing’ universes that are totally causally disjoint from our own, and how that could come into being, particularly given two important features. Firstly, specifying the geometry of a generic universe requires an infinite amount of information because the quantities in $\mathcal{P}_{\text{geometry}}$ are fields on spacetime, in general requiring specification at each point (or equivalently, an infinite number of Fourier coefficients) - they will almost always not be algorithmically compressible. This greatly aggravates all the problems regarding infinity and the ensemble. Only in highly symmetric cases, like the FLRW solutions, does this data reduce to a finite number of parameters. One can suggest that a statistical description would suffice, where a finite set of numbers describe the statistics of the solution, rather than giving a full description. Whether this suffices to adequately describe an ensemble where ‘all that can happen, happens’ is a moot point. We suggest not, for the simple reason that there is no guarantee that all possible models will obey any known statistical description. That assumption is a major restriction on what is assumed to be possible.

Secondly, many universes in the ensemble may themselves have infinite spatial extent and contain an infinite amount of matter, with the paradoxical conclusions that entails (Ellis and Brundrit 1979). To conceive of physical creation of an infinite set of universes (most requiring an infinite amount of information for their prescription, and many of which will themselves be spatially infinite) is at least an order of magnitude more difficult than specifying an existent infinitude of finitely specifiable objects.

The phrase ‘everything that can exist, exists’ implies such an infinitude, but glosses over all the profound difficulties implied. One should note here particularly that problems arise in this context in terms of the continuum assigned by classical theories to physical quantities and indeed to spacetime itself. Suppose for example that we identify corresponding times in the models in an ensemble and then assume that *all* values of the density parameter occur at each spatial point at that time. Because of the real number continuum, this is an uncountably infinite set of models – and genuine existence of such an uncountable infinitude is highly problematic. But on the other hand, if the set of realised models is either finite or countably infinite, then almost all possible models are not realised – the ensemble represents a set of measure zero in the set of possible universes. Either way the situation is distinctly uncomfortable. However we might try to argue around this by a discretization argument: maybe differences in some parameter of less than say 10^{-10} are unobservable, so we can replace the continuum version by a discretised one, and perhaps some such discretisation is forced on us by quantum theory. If this is the intention, then that should be made explicit. That solves the ‘ultraviolet divergence’ associated with the small-scale continuum, but not the ‘infrared divergence’ associated with supposed infinite distances, infinite times, and infinite values of parameters describing cosmologies.

³One way out would be, as quite a bit of work in quantum cosmology seems to indicate, to have time originating or emerging from the quantum-gravity dominated primordial substrate only “later.” In other words, there would have been a “time” or an epoch before time as such emerged. Past time would then be finite, as seems to be demanded by philosophical arguments, and yet the timeless primordial state could have lasted “forever,” whatever that would mean. This possibility avoids the problem of constructibility.

4 Ensembles of FLRW Universes

Having established the broad set of issues concerning multiverses that we believe need to be addressed, we shall for the remainder of this paper limit ourselves to the FLRW sector $\mathcal{M}_{\text{FLRW}}$ of the ensemble of all possible universes \mathcal{M} in order to illustrate these issues⁴. We assume the family considered is filled with matter components characterised by a γ -law equation of state, and mainly restrict our attention to their cosmological parameters, although full consideration of anthropic issues would be characterised by including all the other parameters. Our descriptive treatment will consider FLRW universe domains (whether a true multiverse or separate domains in a single spacetime) as distinct but with common physical characteristics.

4.1 Properties of FLRW models

FLRW models are homogeneous and isotropic models described by the metric

$$ds^2 = -dt^2 + a^2(t) \left(\frac{r^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (15)$$

where $d\Omega^2 = d\vartheta^2 + \sin^2(\vartheta) d\varphi^2$ denotes the line element on the two-dimensional unit sphere, $a(t)$ is the scale-factor, and

$$k = \begin{cases} 1 & \text{for closed models} \\ 0 & \text{for flat models} \\ -1 & \text{for open models} \end{cases}$$

is the normalised curvature. The FLRW model is completely determined by k and the scale-factor $a(t)$, which incorporates the time-evolution and is obtained from the Einstein-Field equations together with the matter description.⁵

Assuming gravity is described by the Einstein field equations, the evolution of FLRW models is described by the Friedmann equation

$$H^2(\Omega - 1) = \frac{k}{a^2}, \quad (16)$$

where $H \equiv \dot{a}/a$ (a dot denotes differentiation with respect to proper time) is the Hubble parameter and Ω the density parameter. We restrict our discussion to models with only a cosmological constant Λ and one matter component which obeys a γ -law equation of state, i.e., its pressure p and density ρ are related by $p = (\gamma - 1)\rho$, where γ is constant. This specification of parameters $p_3(i)$ includes in particular the case of dust ($\gamma = 1$) and radiation ($\gamma = 4/3$). The total density parameter is

$$\Omega = \Omega_m + \Omega_\Lambda. \quad (17)$$

where the matter density parameter is $\Omega_m \equiv \frac{\kappa\rho}{3H^2}$ and the vacuum-energy density parameter is $\Omega_\Lambda \equiv \frac{\Lambda}{3H^2}$ (representing a cosmological constant). These form the parameters $p_4(i)$.

⁴Many discussions implicitly suggest that this is the whole possibility space, as they only consider FLRW models as possibilities. However these clearly form a very small subspace of all geometrical possibilities.

⁵The way the tetrad description given above relates to FLRW universes is described in detail in Ellis and MacCallum (1968); the standard coordinates given here are more convenient if one discusses only the FLRW models.

The second time derivative of the scale factor is determined by the Raychaudhuri equation

$$2q = (3\gamma - 2)\Omega_m - \Omega_\Lambda, \quad (18)$$

where $q \equiv -\frac{\ddot{a}}{aH^2}$ is the dimensionless deceleration parameter. The matter evolution is given by the energy-conservation equation

$$\dot{\rho} = -3\gamma\rho H \quad (19)$$

or equivalently by

$$\dot{\Omega}_m = \Omega_m H (q + 1 - 3\gamma). \quad (20)$$

Besides the normalised curvature k there are two constants of motion, $\chi \equiv \kappa\rho a^{3\gamma}/3$ and the cosmological constant Λ . Given these parameters, the dynamical evolution is determined from the implied initial conditions: $\{a(t_0), \Omega_{i0}, \gamma_i, k\} \Rightarrow a(t)$.

4.2 Parametrising FLRW models

In order to define a FLRW ensemble we need a set of independent parameters which uniquely identify all possible models. We want to consider all possible FLRW models with the same physical laws as in our universe, but possibly different coupling constants. There is then one set of parameters $p_2(i)$ which defines the gravitational “physics” of the model in terms of the coupling constants – for simplicity let us only consider the gravitational constant G here – and further sets $p_5(i), p_4(i)$ which identify the geometry and matter content of the actual model, and which are related to each other via the Einstein field equations.

Among the various options there are two particularly useful parametrisations. Ehlers and Rindler (1989) developed a parametrisation in terms of the observable density parameters (they also include a radiation component) and the Hubble parameter. With $\Omega_k \equiv \frac{k}{H^2 a^2}$ the Friedmann equation becomes

$$\Omega_m + \Omega_\Lambda - 1 = \Omega_k. \quad (21)$$

The curvature parameter Ω_k determines $k = \text{sgn}(\Omega_k)$. For $k \neq 0$ the scale-factor, and hence the metric (15), is determined by

$$a^2(t) = \frac{k}{H^2 \Omega_k} = \frac{k}{H^2 (\Omega_m + \Omega_\Lambda - 1)},$$

while for $k = 0$ its value is unimportant because of scale-invariance in that case. Hence any *state* is completely described by Ω_m, Ω_Λ , and H .

In order to parametrise the models rather than the states, we need to select one particular time t_0 for each model at which we take the above parameters as representative parameters $\Omega_{m0}, \Omega_{\Lambda0}$, and H_0 for this model.⁶ We note that this time t_0 can be model dependent because not all models will reach the age t_0 .

All big-bang FLRW models start with an infinite positive Hubble parameter whose absolute value reaches or approaches asymptotically a minimum value H_{\min} . Hence we could define the time t_0 as the time when the model first takes a certain value $H_0(p_I) > H_{\min}(p_I)$, where p_I represents the model parameters. One particular choice of $H_0(p_I)$ is given by

$$H_0(p_I) \equiv \exp(H_{\min}(p_I))$$

⁶Hence, in general different models correspond to the same values of Ω_m and Ω_Λ , depending on the value of H .

On the other hand, by setting $H_0(p_I) = \text{const}$ and excluding all models which never reach this value one finds easily a parametrisation of all models which reach this Hubble value during their evolution.

While above choice of parameters give a convenient parametrisation in terms of observables which covers closed, flat, and open models, it is disturbing that for each model an arbitrary time has to be chosen. This also leads to a technical difficulty, because the parameters $\{H_0, \Omega_{m0}, \Omega_{\Lambda 0}\}$ are subject to the constraint $H_0 = H_0(p_I)$.

For these reasons it is often convenient to use a set of parameters which are comoving in the state-space, i.e., parameters which are constants of motion. As mentioned above, for open and closed models such a set is given by the matter constant χ , the cosmological constant Λ , and the normalised curvature constant k . For flat models one can rescale the scale factor, which allows us to set $\chi = 1$.

These parameters are related to the observational quantities by (for $k = \pm 1$)

$$\chi = \frac{\Omega_m H^2}{(H^2 |1 - \Omega_m - \Omega_\Lambda|)^{3\gamma/2}} \quad \text{and} \quad \Lambda = 3H^2 \Omega_\Lambda.$$

The evolution of these models through state space is illustrated here in terms of two different parametrisations of the state space, see Figures 1a and 1b. For a detailed investigation of these evolutions for models with non-interacting matter and radiation, see Ehlers and Rindler (1989).

4.3 The possibility space

The structures defined so far are the uniform structures across the class of models in this possibility space, characterised both by laws of physics (in particular General Relativity) and by a restricted class of geometries. It is clear that universes in a multi-universe should be able to differ in at least some properties from each other. We have just characterised the geometrical possibilities we are considering. The next question is, which physical laws and parameters can vary within the ensemble, and which values can they take? For this simplified discussion let us just assume that only the gravitational constant G and the cosmological constant Λ (which also qualifies as a model parameter) are variables, with the ranges⁷ $G \in [0, \infty)$ and $\Lambda \in (-\infty, \infty)$. However, if we consider “all that is possible” within this restricted class of FLRW models, maybe we should consider $G \in (-\infty, \infty)$. There is still considerable uncertainty as to the nature of an ensemble even within this restricted context. Whatever is chosen here defines the set of possibilities that can arise.

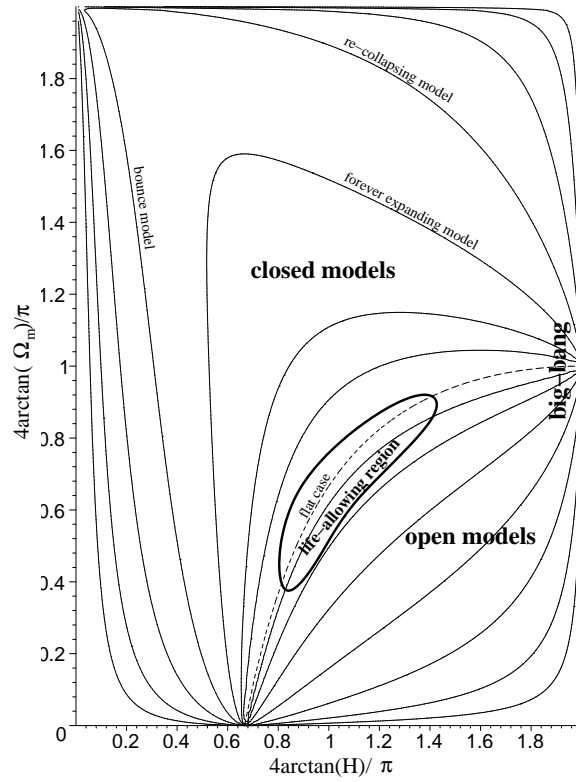
4.4 The measure

For a complete probabilistic description of an ensemble we need not only a distribution function P , but also a measure π for the parameter space (see Section 3.1). The information entropy

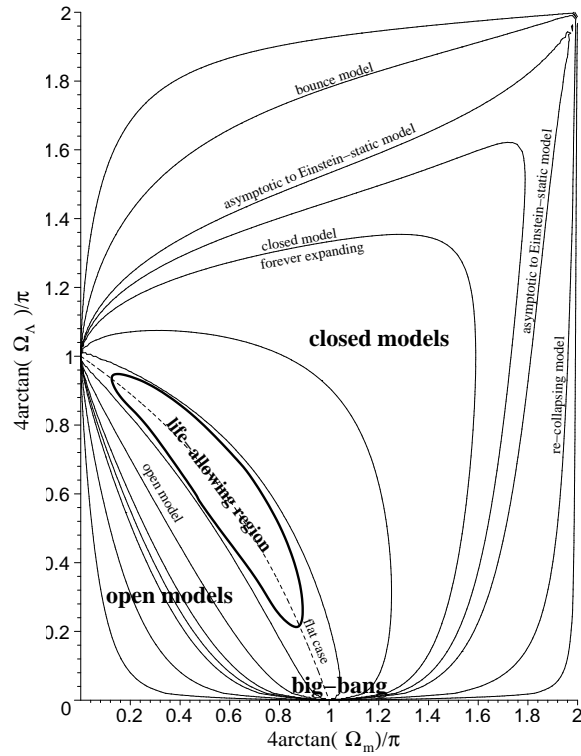
$$S \equiv - \int dx P(x) \log \left(\frac{P(x)}{\mu(x)} \right) \quad (22)$$

is then maximised for the probability distribution equal to this measure, representing the state of minimal knowledge.

⁷It is worth noting that when $G = 0$, we *do* obtain FLRW solutions to the Einstein-Field equations: the Milne universe which is effectively empty – no gravity effective.



(a) $\Omega_m - H$



(b) $\Omega_\Lambda - \Omega_m$ - plane

Without knowledge of the creation mechanism it is impossible to determine this measure with certainty. Nevertheless, we might ask what our best guess for such a measure should be in a state of minimal information, where only a certain set of independent parameters, describing the ensemble, and their ranges are known.

The only known method for constructing such a measure is Jaynes' principle. Its application to FLRW models with γ -law equation of state has been discussed in Kirchner and Ellis (2003). One identifies a set of transformations $x'(x)$ in possibility space which leaves the mathematical structure invariant, and demands that the measure is invariant under these transformations. The two most important cases are given by parameters which can take all real values, and those that take on all positive real values.

In the first case, if z is a valid parameter value, so is $z' = z + \alpha$ for all real α . According to Jaynes' principle the measure should obey $\pi = \mu(z)dz = \mu(z')dz'$ and hence $\mu(z) = \mu(z')$, i.e., the measure is constant.

If on the other hand a parameter u only takes non-zero positive values, we can generate another valid parameter value by $u' = \lambda u$, where $\lambda \in R^+$. Demanding invariance of the measure yields $\pi \propto \frac{1}{u} du$.

These transformations are not unique, and hence one could find many different measures. Nevertheless, in the state of minimum information we don't know what the natural parametrisation is for the possibility space and different measures correspond to different guesses. Surprisingly Jaynes' principle is "relatively invariant" under simple parametrisation changes. For example, introducing a new parametrisation for a positive quantity G by $G = m^n$ for positive m , or by $G = \exp(\lambda)$ for real λ , will give the same measure.

There are two important points to note. Firstly the measure is derived from the chosen set of parameters. Generally a different choice of parameters yields a different minimum-information measure, predicting another maximum-entropy distribution function. Let us consider the example of an ensemble of dust-FLRW models. The different open and closed models are most conveniently parametrised by the constants of motion, which are given by the cosmological constant Λ and $\chi \equiv a\rho^{3\gamma}$, where ρ is the energy density. This leads to the minimum-information measure (Kirchner and Ellis 2003).

$$\pi \propto \frac{d\Lambda d\chi}{\sqrt{\chi}}. \quad (23)$$

Considering dust models ($\gamma = 1$) and the subset of all big-bang models which reach a certain Hubble parameter H_0 at a time t_0 during their evolution this measure becomes

$$\pi \propto \sqrt{\frac{\Omega_{m0}}{|\Omega_0 - 1|^{3/2}}} \left| \frac{1}{\Omega_{m0}} - \frac{3/2}{\Omega_0 - 1} \right| d\Omega_{m0} d\Omega_{\Lambda 0},$$

with $\Omega_{\Lambda 0} \leq 1 + \Omega_{m0}/2$. On the other hand, as mentioned above, there is a convenient parametrisation for this particular subset of models (Ehlers and Rindler 1989) in terms of the observables Ω_{m0} and $\Omega_{\Lambda 0}$ (in Ehlers and Rindler (1989) an additional radiation component was also included). Using this parametrisation yields the minimum information measure

$$\pi \propto \frac{1}{\sqrt{\Omega_{m0}}} d\Omega_{m0} d\Omega_{\Lambda 0},$$

which is clearly different from the above result.

Secondly, the measure is in general non-normalisable and hence there is no normalisable maximum-entropy distribution. Without additional information we are not

able to calculate certain probabilities. Since it seems questionable whether there will ever be additional information about the ensemble of universes available, one has to accept that certain questions will have no well defined probabilities.

It should be mentioned that we encounter similar problems when we want to find a probability measure for physical parameters like the gravitational constant G . Let us assume that G can take any non-zero positive value. Jaynes' principle then suggests the probability measure $\pi_G \propto \frac{dG}{G}$. On the other hand, if we decide to use ⁸ $m = \sinh(G)$ as our parameter, then we find the different measure

$$\pi = \frac{dm}{m} = \frac{\cosh(G)}{\sinh(G)} dG.$$

4.5 Distribution Functions on $\mathcal{M}_{\text{FLRW}}$

Now, having properly parametrised $\mathcal{M}_{\text{FLRW}}$ and defined a measure on it, we can represent particular multiverses by giving distribution functions over the parameter-space (as discussed in Section 3.4). Given a distribution function f it determines the number of universes in a small parameter-interval by

$$dN = f(p_I)\pi,$$

which is invariant under a change of parametrisation. Hence it is the combination of measure and distribution function which is of importance.

While distribution functions can be parametrised by any set of coordinates over the possibility space, we need different distribution functions for different possibility spaces. For example, if universes in a multiverse *must* have a common value for the gravitational constant G then distribution functions must not depend on G .

It is clear that a particular distribution function can be expressed in any set of coordinates. Obviously there is a vast set of possible distribution functions. We want to examine some particular examples.

Firstly, one could have a distribution function which is constant over the parameter-space. The actual ensemble then really depends on the measure and is the maximum-entropy distribution (which maximises (22)⁹). If we choose the observational quantities $H, \Omega_m, \Omega_\Lambda$ to represent the model and allow for different values of the gravitational coupling constant G this would be

$$f(H, \Omega_m, \Omega_\Lambda, G) = \text{const.}$$

for all allowed values of the stated parameters. On the other hand, if we choose the constants of motion as coordinates in possibility space

$$f(k, \chi, \Lambda, G) = \text{const.}$$

The probability $P_{\mathcal{A}}$ to find a universe in a certain parameter-region \mathcal{A} is given by

$$P_{\mathcal{A}} = \frac{\int_{\mathcal{A}} f(k, \chi, \Lambda, G)\pi.}{\int f(k, \chi, \Lambda, G)\pi.},$$

where the integral in the denominator extends over the whole possibility space. For many distribution functions, like for the above constant distribution function together

⁸However, it should be noted that any power ($m = G^n, m \in +, n \neq 0$) and logarithmic relationship ($m = \ln(G), m \in$) leads to the same measure $d\mu_G$.

⁹In (22) $P(x)dx$ corresponds to $f(p_I)\pi$.

with (23), this expression is not well defined. Let us assume that the measure is non-integrable, i.e., non-normalisable, and that we have a constant distribution function. If the set \mathcal{A} does not include any point of non-integrability of the measure then $P_{\mathcal{A}} = 0$, if it includes all points of non-integrability then $P_{\mathcal{A}} = 1$, but if it includes only some of the non-integrabilities then $P_{\mathcal{A}}$ is not well defined.

Of course, the above expression might be integrable for all sets \mathcal{A} given a good distribution function. For instance the distribution function

$$f(k, \chi, \Lambda) \propto \exp(-\chi - \Lambda^2)$$

together with the measure (23) is integrable everywhere. On the other hand

$$f(k, \chi, \Lambda) \propto \frac{\exp(-\Lambda^2)}{\sqrt{\chi + 1}}$$

diverges for $\chi \rightarrow \infty$. A distribution function can also introduce an additional divergence, for instance

$$f(k, \chi, \Lambda) \propto \frac{\exp(-\Lambda^2)}{\sqrt{\chi}}$$

is non-integrable at $\chi = 0$ and $\chi \rightarrow \infty$.

A multiverse might contain a finite or infinite countable number of universes. In these cases the distribution function contains Dirac δ -functions, e.g.,

$$f(k, \chi, \Lambda) = \begin{cases} \sqrt{\chi} 5 \delta^2(\chi - 3, \Lambda - .5) & \text{for } k = 1 \\ 2 \delta(\Lambda - .7) & \text{for } k = 0 \\ \sqrt{\chi} 3 \delta^2(\chi - .1, \Lambda - 2) & \text{for } k = -1 \end{cases}$$

which represents a multiverse which contains 5 copies of closed FLRW models with $\chi = 3$ and $\Lambda = .5$, etc. The distribution function

$$f(k, \chi, \Lambda) = \begin{cases} \sqrt{\chi} \sum_{i=1}^{\infty} \delta(\chi - 1/i, \Lambda - 1) & \text{for } k = 1 \\ 0 & \text{for } k \neq 1 \end{cases}$$

represents an ensemble with a countably infinite number of universes – all are closed with $\Lambda = 1$ and one for each $\chi = 1/i$. Similarly one could imagine an ensemble of 10^7 copies of our universe, which would be represented by the distribution function

$$f(k, \chi, \Lambda) = 10^7 \sqrt{\chi} \delta_k^{k_0} \delta(\chi - \chi_0) \delta(\Lambda - \Lambda_0),$$

where k_0, χ_0, Λ_0 represent the parameter values for our universe. This is very unlikely in terms of a generating mechanism, but for ensembles without generating mechanisms it is as likely as any other possibility. If a multiverse is “tested” by its prediction that our universe is a likely member, then such an ensemble should be the most satisfying one – but then we might just as well be happy with one copy, i.e., just our universe.

Similar distribution functions determine the distribution of physical parameters like the gravitational constant G . For example with $G \in \mathbf{R}$ the minimum information measure is $d\mu_G = dG$ and

$$f(G) = \exp(-G^2)$$

gives a Gaussian distribution around $G = 0$. If, on the other hand $G \in (0, \infty)$ the measure is dG/G and $f(g) = G \exp(-G)$ would be an example of a distribution function.

One can imagine various types of distributions, e. g., a Gaussian distribution in G or in H_0 , or in the other parameters. But, in order to establish these in a non-arbitrary way, we need a theory of how this particular ensemble is selected for from all the other possible ones.

4.6 The problem of infinities again

Even within the restricted set of FLRW models, one of the most profound issues is the problem of realised infinities: if all that is possible in this restricted subset happens, we have multiple infinities of realised universes in the ensemble. First, there are an infinite number of possible spatial topologies in the negative curvature case, so an infinite number of ways that universes which are locally equivalent can differ globally. Second, even though the geometry is so simple, the uncountable continuum of numbers plays a devastating role locally: is it really conceivable that FLRW universes actually occur with *all* values independently of both the cosmological constant and the gravitational constant, and also all values of the Hubble constant at the instant when the density parameter takes the value 0.97? This gives 3 separate uncountably infinite aspects of the ensemble of universes that is supposed to exist. The problem would be allayed if spacetime is quantized at the Planck level, as suggested for example by loop quantum gravity. In that case one can argue that all physical quantities also are quantized, and the uncountable infinities of the real line get transmuted into finite numbers in any finite interval – a much better situation. We believe that this is a physically reasonable assumption to make, thus softening a major problem for many ensemble proposals. But the intervals are still infinite for many parameters in the possibility space. Reducing the uncountably infinite to countably infinite does not in the end resolve the problem of infinities in these ensembles. It is still an extraordinarily extravagant proposal.

4.7 The anthropic subset

We can identify those FLRW universes in which the emergence and sustenance of life is possible at a broad level¹⁰ – the necessary cosmological conditions have been fulfilled allowing existence of galaxies, stars, and planets if the universe is perturbed, so allowing a non-zero factor $\Pi = P_{gal} * R * f_S * f_p * n_e$ as discussed above. These are indicated in the Figures above (anthropic universes are those intersecting the regions labelled “life allowing”). The fraction of these that will actually be life-bearing depends on the fulfilment of a large number of other conditions represented by the factor $F = f_l * f_i$, which will also vary across a generic ensemble, and the above assumes this factor is non-zero..

5 On the origin of ensembles

Ensembles have been envisaged both as resulting from a single causal process, and as simply consisting of discrete entities. We discuss these two cases in turn, and then show that they are ultimately not distinguishable from each other.

5.1 Processes Naturally Producing Ensembles

Over the past 15 or 20 years, many researchers investigating the very early universe have proposed processes at or near the Planck era which would generate a really existing ensemble of expanding universe domains, one of which is our own observable universe. In fact, their work has provided both the context and stimulus for our discussions in this paper. Each of these processes essentially selects a really existing

¹⁰More accurately, perturbations of these models can allow life – the exact FLRW models themselves cannot do so.

ensemble through a generating process from a set of possible universes, and often lead to proposals for a natural definition of a probability distribution on the space of possible universes. Here we briefly describe some of these proposals, and comment on how they fit within the framework we have been discussing.

Andrei Linde's (1983, 1990) chaotic inflationary proposal (see also Linde (2003) and references therein) is one of the best known scenarios of this type. The scalar field (inflaton) in these scenarios drives inflation and leads to the generation of a large number of causally disconnected regions of the Universe. This process is capable of generating a really existing ensemble of expanding FLRW-like regions, one of which may be our own observable universe region, situated in a much larger universe that is inhomogeneous on the largest scales. No FLRW approximation is possible globally; rather there are many FLRW-like sub-domains of a single fractal universe. These domains can be very different from one another, and can be modelled locally by FLRW cosmologies with different parameters.

Linde and others have applied a stochastic approach to inflation (Starobinsky 1986, Linde, *et al.* 1994, Vilenkin 1995, Garriga and Vilenkin 2001, Linde 2003), through which probability distributions can be derived from inflaton potentials along with the usual cosmological equations (the Friedmann equation and the Klein-Gordon equation for the inflaton) and the slow-roll approximation for the inflationary era. A detailed example of this approach, in which specific probability distributions are derived from a Langevin-type equation describing the stochastic behaviour of the inflaton over horizon-sized regions before inflation begins, is given in Linde and Mezhlumian (2003) and in Linde *et al.* (1994). The probability distributions determined in this way generally are functions of the inflaton potential.

This kind of scenario suggests how overarching physics, or a "law of laws" (represented by the inflaton field and its potential), can lead to a really existing ensemble of many very different FLRW-like regions of a larger Universe. However these proposals rely on extrapolations of presently known physics to realms far beyond where its reliability is assured. They also employ inflaton potentials which as yet have no connection to the particle physics we know at lower energies. And these proposals are not directly observationally testable – we have no astronomical evidence the supposed other FLRW-like regions exist. Thus they remain theoretically based proposals rather than established fact. There additionally remains the difficult problem of infinities: eternal inflation with its continual reproduction of different inflating domains of the Universe is claimed to lead to an infinite number of universes of each particular type (Linde, private communication). How can one deal with these infinities in terms of distribution functions and an adequate measure? As we have pointed out above, there is a philosophical problem surrounding a realised infinite set of any kind.

Finally, from the point of view of the ensemble of all possible universes often invoked in discussions of multiverses, all possible inflaton potentials should be considered, as well as all solutions to all those potentials. They should all exist in such a multiverse, which will include chaotic inflationary models which are stationary as well as those which are non-stationary. Many of these potentials may yield ensembles which are uninteresting as far as the emergence of life is concerned, but some will be bio-friendly. The price of this process for creating anthropically favourable universe regions is the multiplication of realised infinities, most of which will be uncountable (for example the parameters in any particular form of inflaton potential will take all possible values in an interval of real numbers).

5.2 Probability distributions for the cosmological constant

Weinberg (2000) and Garriga and Vilenkin (2001) derive a probability distribution for the cosmological constant in the context of an ensemble of regions generated in the same inflationary sequence via the action of a given inflaton potential where the cosmological constant is given by the potential energy of a scalar-field. In multi-domain universes, where spatial variations in a scalar-field cause different regions to inflate at different rates, the cosmological constant should be distributed according to some probability distribution $P(\rho_\Lambda)$. During inflation the scalar field undergoes randomisation by quantum fluctuations, such that later on its values in different regions are distributed according to “the length” in field space (Garriga and Vilenkin 2002). This leads to a probability distribution (or distribution function – the probability distribution is just the normalised distribution function) of values of the vacuum-energy density ρ_Λ in these regions given by

$$P(\rho_\Lambda)d\rho_\Lambda \propto \frac{d\rho_\Lambda}{|V'(\phi)|},$$

where $V(\phi)$ is the inflaton potential, and the prime signifies differentiation with respect to the inflaton ϕ .

It has been suggested (Vilenkin 1995, Weinberg 1997) that the way the probability distribution for existence of galaxies depends on the cosmological constant can be approximated by

$$P_{gal} = N(\rho_\Lambda)P(\rho_\Lambda) \tag{24}$$

where $N(\rho_\Lambda)$ is the fraction of baryons that form galaxies. The requirement of structure formation as a pre-requisite for life places strong anthropic constraints on the domains in which observers could exist; these constraints must be satisfied in the really existing universe.

Let us first note that galaxy formation is only possible for a narrow range around $\rho_\Lambda = 0$ (Weinberg 2000). It has been shown that anthropic restrictions demand $\rho_\Lambda \lesssim 10^{-28} \frac{\text{g}}{\text{cm}^3}$ (Kallosh and Linde 2002, Garriga and Vilenkin 2002). Consequently the anthropic selection factor $N(\rho_\Lambda)$ is sharply peaked and vanishes for $|\rho_\Lambda| > \rho_{\Lambda \text{ max}}$ for some $\rho_{\Lambda \text{ max}}$, which is of the same order of magnitude as the observed cosmological constant. In scalar-field models $P(\rho_\Lambda)$ is in direct relation to the *a priori* distribution of the scalar-field fluctuations and it has been argued (Weinberg 2000) that for a wide class of potentials the variations of $P(\rho_\Lambda)$ over the anthropically allowed range (where $N(\rho_\Lambda) \neq 0$) should be negligible. Nevertheless, as has been shown in (Vilenkin-Garriga 2002) this is not always the case, in particular for power-law potentials $V(\phi) = \phi^n$ with $n > 1$ one finds an integrable divergence at $\rho_\Lambda = 0$.

It is clear that a similar relation to (24) should hold for multiverses in the wider sense. Nevertheless, one could imagine multiverses containing universes with and without scalar-field, or with different potentials. Hence we cannot link the distribution of the cosmological constant to that of the scalar-field in a unique way, and there is a vast choice for possible *a priori* probability distributions for the cosmological constant. Let us assume that the cosmological constant is a remnant of some underlying (unknown) theory and as such might be restricted to some domain of values. Depending on this domain one finds different possible minimum information measures, which result from Jaynes’ principle. If the domain is given by all real numbers then the (non-normalisable) measure will be constant. If on the other hand the domain is given by all positive real numbers then the minimum-information measure gives an

(non-normalisable) *a priori* probability distribution proportional to $1/\rho_\Lambda$ (Kirchner and Ellis 2003). In this case the divergence is located inside the anthropically allowed region and is non-integrable. For this case the expectation value vanishes, i.e.,

$$\bar{\rho}_\Lambda = \frac{\int_0^{\rho_\Lambda \max} \rho_\Lambda \frac{1}{\rho_\Lambda} d\Lambda}{\int_0^{\rho_\Lambda \max} \frac{1}{\rho_\Lambda} d\Lambda} = 0$$

and we fail to explain the observed non-zero value of the cosmological constant.

An interesting alternative is given by allowing the cosmological constant to take values in the domain $R^+ \cup \{0\}$ (e.g., if the cosmological constant prediction is given by a quadratic term). The minimum information-measure is then proportional to $1/\sqrt{\rho_\Lambda}$. Again there is a divergence in the anthropically allowed region, but this time it is integrable. The expectation value becomes

$$\bar{\rho}_\Lambda = \frac{\int_0^{\rho_\Lambda \max} \rho_\Lambda \frac{1}{\sqrt{\rho_\Lambda}} d\Lambda}{\int_0^{\rho_\Lambda \max} \frac{1}{\sqrt{\rho_\Lambda}} d\Lambda} = \frac{1}{3} \rho_\Lambda \max.$$

5.3 The existence of regularities

Consider now a genuine multiverse. Why should there be any regularity at all in the properties of universes in such an ensemble, where the universes are completely disconnected from each other? If there are such regularities and specific resulting properties, this suggests a mechanism creating that family of universes, and hence a causal link to a higher domain which is the seat of processes leading to these regularities. This in turn means that the individual universes making up the ensemble are not actually independent of each other. They are, instead, products of a single process, as in the case of chaotic inflation. A common generating mechanism is clearly a causal connection, even if not situated in a single connected spacetime – and some such mechanism is needed if all the universes in an ensemble have the same class of properties, for example being governed by the same physical laws or meta-laws.

The point then is that, as emphasized when we considered how one can describe ensembles, any multiverse with regular properties that we can characterise systematically is necessarily of this kind. If it did not have regularities of properties across the class of universes included in the ensemble, we could not even describe it, much less calculate any properties or even characterise a distribution function.

Thus in the end the idea of a completely disconnected multiverse with regular properties but without a common causal mechanism of some kind is not viable. There must necessarily be some pre-realisation causal mechanism at work determining the properties of the universes in the ensemble. What are claimed to be totally disjoint universes must in some sense indeed be causally connected together, albeit in some pre-physics or meta-physical domain that is causally effective in determining the common properties of the multiverse.

Related to this is the issue that we have emphasized above, namely where does the possibility space come from and where does the distribution function come from that characterises realised models? As emphasized above, we have to assume that some relevant meta-laws pre-exist. We now see that we need to explain also what particular meta-laws pre-exist. If we are to examine ‘all that might be, exists’, then we need to look at the ensemble of all such meta-laws and a distribution function on this set. We seem to face an infinite regress as we follow this logic to its conclusion, and it is not clear how to end it except by arbitrarily calling a stop to this process. But then we have not looked at all conceivable possibilities.

6 Testability and Existence

Finally, the issue of evidence and testing has already been briefly mentioned. This is at the heart of whether an ensemble or multiverse proposal should be regarded as physics or as metaphysics.

6.1 Evidence and existence

Given all the possibilities discussed here, which specific kind of ensemble is claimed to exist? Given a specific such claim, how can one show that this is the particular ensemble that exists rather than all the other possibilities?

There is no direct evidence of existence of the claimed other universe regions, nor can there be any, for they lie beyond the visual horizon; most will even be beyond the particle horizon, so there is no causal connection with them; and in the case of a true multiverse, there is not even any possibility of any indirect causal connection of any kind - the universes are then completely disjoint and nothing that happens in any one of them is linked to what happens in any other one.

What weight does a claim of such existence carry, in this context when no direct observational evidence can ever be available? The point is that there is not just an issue of showing a multiverse exists - if this is a scientific proposition one needs to be able to show which specific multiverse exists; but there is no observational way to do this. Indeed if you can't show which particular one exists, it is doubtful you have shown any one exists. What does a claim for such existence mean in this context?

These issues are discussed in more depth in the accompanying philosophical paper, where we consider the various ways one may claim entities exist even when there is no direct or even indirect evidence for such existence. One ends up in deep philosophical waters. That is unavoidable if one is to seriously argue the claim for existence of a multiverse. Even the concept of what 'existence' might mean in this context needs careful consideration.

6.2 Observations and Physics

The one way one might make a reasonable claim for existence of a multiverse would be if one could show its existence was a more or less inevitable consequence of well-established physical laws and processes. Indeed, this is essentially the claim that is made in the case of chaotic inflation. However the problem is that the proposed underlying physics has not been tested, and indeed may be untestable. There is no evidence that the postulated physics is true in this universe, much less in some pre-existing metaspaces that might generate a multiverse. Thus belief in the validity of the claimed physics that could lead to such consequences is just that, a belief - it is based on unproved extrapolation of established physics to vastly beyond where it has been tested. The issue is not just that the inflaton is not identified and its potential untested by any observational means - it is also that, for example, we are assuming quantum field theory remains valid far beyond the domain where it has been tested, and we have faith in that extreme extrapolation despite all the unsolved problems at the foundation of quantum theory, the divergences of quantum field theory, and the failure of that theory to provide a satisfactory resolution of the cosmological constant problem.

6.3 Observations and probabilities

The ‘doomsday argument’ has led to a substantial literature on relating existence of universe models to evidence, based on analysis of probabilities, often using a model of choosing a ball randomly from an urn, and of associated selection effects (see e.g. Bostrom 2002). However usually these models either in effect assume an ensemble exists, or else are content to deal with potentially existing ensembles rather than actually existing ones (see e.g. Olum 2002). That does not deal with the case at hand. One would have to extend those arguments to trying to decide, on the basis of a single ball drawn from the urn, as to whether there was one ball in the urn or an infinite number. It is not clear to us that the statistical arguments used in those papers leads to a useful conclusion in this singular case, which is the case of interest for the argument in this paper.

In any case, in the end those papers all deal just with observational probabilities, which are never conclusive. Indeed the whole reason for the anthropic literature is precisely the fact that biophilic universes are clearly highly improbable within the set of all possible universes (see e.g. the use of Anthropic arguments as regards the value of Λ referred to in Section 5.2). We are working in a context where large improbabilities are the order of the day. Indeed that is why multiverse concepts were introduced in the first place - to try to introduce some form of scientific explanation into a context where the probabilities of existence of specific universe models preferred by observation are known to be very small.

6.4 Observations and disproof

Despite the gloomy prognosis given above, there are some specific cases where the existence of a chaotic inflation (multi-domain) type scenario can be disproved. These are when we live in a ‘small universe’ where we have already seen right round the universe (Ellis and Schreiber 1986, Lachieze-Ray and Luminet 1995) for then the universe closes up on itself in a single FLRW-like domain and so no further such domains that are causally connected to us in a single connected spacetime can exist.

This ‘small universe’ situation is observationally testable, and indeed it has been suggested that the CBR power spectrum might already be giving us evidence that this is indeed so, because of its lack of power on the largest angular scales (Luminet et al, 2003). This proposal can be tested in the future by searching for identical circles in the CMB sky. That would disprove the usual chaotic inflationary scenario, but not a true multiverse proposal, for that cannot be shown to be false by any observation. Neither can it be shown to be true.

7 Conclusion

The introduction of the multiverse or ensemble idea is a fundamental change in the nature of cosmology, because it aims to challenge one of the most basic aspects of standard cosmology, namely the uniqueness of the universe (see Ellis 1991, 1999 and references therein). However previous discussions have not made clear what is required in order to define a multiverse, although some specific physical calculations have been given based on restricted low-dimensional multiverses. The aim of this paper is to make clear what is needed in order to properly define a multiverse, and then examine some of the consequences that flow from this.

Our fundamental starting point is the recognition that there is an important distinction to be made between possible universes and realised universes, and our main conclusion is that a really existing ensemble or multiverse is not *a priori* unique, nor uniquely defined. It must somehow be selected for. We have pointed out a clear distinction between an ensemble of possible universes \mathcal{M} , and an ensemble of really existing universes, which is envisioned as generated by the given primordial process or action of an overarching cosmic principle. These effectively select a really existing multiverse from the possibilities in \mathcal{M} , and, as such, effectively define a distribution function over \mathcal{M} . Thus, there is a definite causal connection, or “law of laws”, relating all the universes in these multiverses. It is this really existing ensemble of universes, *not* the ensemble of all possible universes, which provides the basis for anthropic arguments. Anthropic universes lie in a small subset of \mathcal{M} , whose characteristics we understand to some extent. It is very likely that the simultaneous realisation of *all* the conditions for life will pick out only a very small sector of the parameter space of all possibilities: anthropic universes are fine-tuned.

The fine-tuning problem is very controversial. Two counter-attacks maintain that there is no fine-tuning problem, so it is not necessary to construct solutions to it by employing the multiverse idea. The first promotes the view that whatever happens will always be unlikely (any hand of cards is as unlikely as any other). Thus, since it is just an example of chance, there is nothing special about a universe that admits life. The counter response is that the existence of life is quite unlike anything else in the physical world – its coming into being is not just like choosing one out of numerous essentially identical hands of cards. It is like being transformed into an entirely different higher level game, and so does indeed require explanation. The second counter-attack argues that inflation explains the current state of the universe, making its apparently unlikely state probable. However, this move is only partially successful, since very anisotropic or inhomogeneous models may never inflate. The counter response is that this does not matter: however small the chances are, if it works just once then that is sufficient to give a model close enough to the standard FLRW cosmological models to be friendly to life. But this does not account for the rest of the coincidences enabling life, involving particle masses and the values of the fundamental constants. Perhaps progress in quantum cosmology will in the future lead to some unique theory of creation and existence that will guide the discussion. At present, uniqueness eludes us.

Among those universes in which the necessary cosmic conditions for life have been fulfilled is the subset of almost-FLRW universes which are possible models of our own observable universe, given the precision of the observational data we have at present. It is, however, abundantly clear that “really existing multiverses” which can be defined as candidates for the one to which our universe belongs are *not* unique, and neither their properties nor their existence is directly testable. The only way in which arguments for the existence of one particular kind of multiverse would be scientifically acceptable is if, for instance, there would emerge evidence (either direct or indirect) for the existence of specific inflaton potential which would generate one particular kind of ensemble of expanding universe domains.

Despite these problems, the idea of a multiverse is probably here to stay with us - it is an important concept that needs exploration and elucidation. Does the idea that ‘all that can exist, exists’ in the ensemble context provide an explanation for the anthropic puzzles? Yes it does do so. The issue of fine tuning is the statement that the biophilic set of universes is a very small subset of the set of possible universes; but if all that can exist exists then there are universe models occupying this biophilic subspace. However there are the following problems: (i) the issue of realised infinities

discussed above, (ii) the problem of our inability to describe such ensembles because we don't know what all the possibilities are, so our solution is in terms of a category we cannot fully describe, and (iii) the multiverse idea is not testable or provable in the usual scientific sense; existence of the hypothesized ensemble remains a matter of faith rather than proof. Furthermore in the end, it simply represents a regress of causation. Ultimate questions remain: Why this multiverse with these properties rather than others? What endows these with existence and with this particular type of overall order? What are the ultimate boundaries of possibility – what makes something possible, even though it may never be realised? In our view these questions - Issues 1 and 2 discussed in this paper - cannot be answered scientifically because of the lack of any possibility of verification of any proposed underlying theory. They will of necessity have to be argued in philosophical terms.

The concept of a multiverse raises many fascinating issues that have not yet been adequately explored. The discussion given here on how they can be described will be useful in furthering this endeavour.

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