

The Philosophy of the Young Kant



The Precritical Project

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Introduction



SURVEYS OF KANT'S thought often begin with a discussion of the *Critique of Pure Reason*. This work, though, was not the starting point for Kant's philosophical career. By the time he completed the manuscript of the *Critique of Pure Reason* in the spring of 1781, Kant was a relatively old man, a fifty-six-year-old full professor who, had he lived today, would not have been that far away from retirement. He had written his philosophical debut, the *Thoughts on the True Estimation of Living Forces*, as a twenty-two year old university student. In the thirty-five years before the eventual publication of the first *Critique*, Kant would become a prolific philosopher authoring a number of books, treatises, and essays. These works make up Kant's precritical philosophy.

The focus of *The Philosophy of the Young Kant* is Kant's philosophical development from 1746 to 1766, the first two decades of his career. I shall argue that the central theme in this period was Kant's struggle for a coherent philosophy of nature from 1754 to 1766. This was Kant's precritical project, and it turned out to be the most ambitious venture of his life. He attempted to integrate Newtonian physics in a comprehensive and speculative framework that explained the macroscopic features of the universe as well as its microstructure, that accounted for its past as well as for its present, that permitted the copresence of rational freedom and deterministic lawfulness, and that illuminated the relation of God to the world.

The precritical project of a unified philosophy of nature emerged in 1754, eight years after the completion of the major portions of the *Living Forces*. (Kant wrote the *Thoughts on the True Estimation of Living Forces* in 1746, added a dedication and a preface in 1747, and had it published in 1749). The precritical project fell apart in 1766. Although the project spanned only twelve years, it involved Kant's most intense and inspired productivity before the 1780s. The precritical period lasted from 1746 to 1780, but most of the precritical texts were written between 1754 and 1766. Of the thirty precritical texts, twenty-three belong to the phase of the precritical project. Kant published two books during this phase, the *Universal Natural History and Theory of the Heavens* (1755) and *The Only Possible Argument in Support of a Demonstration of the Existence of God* (1763). In addition to numerous papers, he also composed six major treatises, the *New Elucidation* (1755), the *Physical Monadology* (1756), the *Negative Quantities* (1763), the *Observations* (1764), the *Prize Essay* (1764), and the *Dreams of a Spirit-Seer* (1766). After this phase, Kant would write the essay on the *Directions in Space* (1768) and the important *Inaugural Dissertation* (1770). In the ten years that followed, only a book review and three marginal essays appeared. During this so-called silent decade, Kant wrestled with what would become the first *Critique*.

With few exceptions, the works from 1754 to 1766 were stepping stones toward Kant's envisioned reconciliation of metaphysics and natural science. In eighteenth-century German philosophy, metaphysics was an energetic philosophical discipline. Its archetype was Christian Wolff's *Vernünfftige Gedancken von Gott, der Welt und der Seele des Menschen, auch allen Dingen überhaupt* (1719; "Rational Thoughts on God, the World, the Human Soul, and All Things in General"). This book, commonly referred to as the *German Metaphysics*, was an exercise in conjectural exuberance as well as in dogmatic scholasticism. The branches of metaphysics were rational theology, rational psychology, rational cosmology, and ontology. Rational theology involved arguments of God's existence and analyses of God's character; rational psychology was concerned with the constitution of the soul and its interaction with the body; rational cosmology contained imaginative and sweeping vistas of corporeal nature; and ontology consisted largely of variations on the well-trodden themes of contradiction and sufficient reason. For the young Kant, metaphysics was about speculative arguments that address the grand questions of philosophy, such as the existence of a divine being, the immortality and freedom of the soul, and the structure and purpose of the universe. He had not yet identified the synthetic a priori as the hallmark of metaphysical propositions. The actual metaphysical arguments of the day, including Kant's own, comprised empirical generalizations as well as conceptual constructions.

"Natural science" meant physics for the young Kant. Isaac Newton's *Principia* (1686) became his authoritative model of a text in natural science. Kant was not alone with this assessment; in the course of the eighteenth century, Newtonian celestial mechanics emerged as the uncontested paradigm of physics. Natural science, in the sense of a Newtonian physics, amounted to

the use of observation and experimentation to articulate quantifiable laws, describing a regular, predictable, and deterministic system of nature.

In the first half of the eighteenth century, natural science and metaphysics had not yet been divorced. Both were part of what was known as philosophy of nature. As the title of Newton's work already revealed—*Philosophiae Naturalis Principia Mathematica* (“Mathematical Principles of Natural Philosophy”)—Newton offered his findings as the mathematical principles of philosophy of nature. In general, philosophy of nature was about the organization and composition of physical reality. It involved two modes, the quantitative-empirical procedure that was characteristic of Newton's perspective and which would later evolve to natural science, and the qualitative-speculative approach of metaphysics. In the generation before Kant's birth, tensions between these two modes began to fracture their common ground. One fissure appeared during the priority dispute (the controversy over the invention of the calculus). Newton concluded his rebuke of Leibniz, in the *Account of the “Commercium Epistolicum”* (1715), with a condemnation of the metaphysical leanings of the latter. Another crack emerged during the *Pietismusstreit*, the Pietists' quarrel with Wolff. In his *Causa Dei et religionis naturalis adversus atheismum* (1723; “The Case of God and of Natural Religion against Atheism”), the metaphysician and theologian Joachim Lange denounced Wolff's sympathy for a mechanist explanation of nature as blasphemy and materialism. A further fault-line opened in the wake of the success of the third edition of the *Principia* (1729) on the continent. The third edition of Newton's great work caused the effective demolition of the Cartesian theory of the vortices in the 1740s. The theory of universal gravitation was becoming the dominant explanation of the universe. Whereas Cartesian mechanics involved a comfortable mixture of speculative, quantitative, and empirical elements, Newtonian physics projected itself as a more rigorous enterprise. Following the dictum of their master, “I feign no hypotheses,” many Newtonians, in particular in France and Great Britain, tended to be less tolerant of speculative approaches than their Cartesian predecessors.

By mid-century, the tensions between the scientific and the metaphysical approaches to nature had become so visible that the status of physics was anything but clear. Physics still had something to do with philosophy, but it had reached a level of autonomy that distinguished it from straightforward philosophical disciplines such as logic or ontology. Christian August Crusius's *Physica*, that is, the *Anleitung über natürliche Begebenheiten ordentlich und vorsichtig nachzudenken* (1749; “Guide to an Orderly and Careful Reflection on Natural Events”), begins with a discussion of the meaning of physics. Crusius claims that physics, in the wide sense, is a branch of philosophy that deals with the contingent things in the world and, in the narrow sense, is a science that investigates the composition and effects of natural bodies.

In the *Living Forces*, Kant had been quite fond of the approach of the Leibnizian-Wolffian School Philosophy. After this student piece, he grasped the fundamental significance of Newton's work. At the same time, he con-

tinued to sympathize with certain aspects of the systems of the School Philosophers. The scientific perspective, exemplified by Newtonian physics, and the metaphysical viewpoint, embodied in the tomes of the School Philosophers, had now acquired equal relevance for Kant. The uneasy relationship of the two approaches to the investigation of nature defined his task, and it gave the precritical project its direction: a new philosophy of nature needed to be created that would once again succeed as a common ground and that could avert the threatening rift between quantitative-empirical inquiries and qualitative-speculative demonstrations.

The precritical theories have often been dismissed as unworthy of attention—there is the “important” Kant, the author of the three *Critiques*, vital for every student of philosophy, and there is the “immature” Kant, the author of the works preceding the *Critiques*, relevant for antiquarians at best. Kant himself was in no small part responsible for this view. Arguing that his early investigations were metaphysical and thus untenable, he publicly rejected his early writings, discouraged his students from reading them, and urged his first editor to exclude them from a collection of his works. When Kant reviewed the *Ideen zur Philosophie der Geschichte der Menschheit* (1784; “Ideas to a Philosophy of the History of Mankind”) by his former student Johann Gottfried Herder and realized that it had been inspired by his own early views, he reacted with dismissive irony. The aging Kant balanced the frank repudiation of his early writings with a hearty approval of his later works. The discovery of the subjectivity of space and time as a priori forms of intuition implied an ontological dualism between the sensible and the intelligible, which ruled out the notion of the unified nature that the precritical project had presupposed. This discovery struck him as a “great light.” The *Critique of Pure Reason* (1781) was for Kant a “philosophical revolution” that overthrew the precritical efforts for good.¹

The neo-Kantians of the nineteenth century implicitly endorsed Kant’s self-assessment and merely promoted the critical philosophy. Leading Kant scholars of the twentieth century left as their legacy a negative appraisal of the precritical philosophy. Four points of their critique were particularly conspicuous: that the early philosophy lacked originality because the early views were just eclectic blends of Leibnizian-Wolffian and Newtonian ideas; that it lacked continuity because of a clean break between the precritical and the critical periods; that it lacked relevance to the subsequent development of philosophy because it dealt with obsolete issues; and worst of all, that it lacked coherence because Kant underwent an erratic development characterized by sudden reversals of opinion. As Lewis White Beck put it, Kant prior to the critical philosophy “would deserve a quarter of a page in *Überweg*.”² Most people, it seems, agree: of the 500-odd articles on Kant that appeared in the *Kant-Studien* in the last sixty years, less than two dozen are about the precritical philosophy.³

I think that such a dismissal of the precritical period is implausible, not the least because it suggests an overly black and white picture of Kant’s intellectual life: a mediocre thinker until his mid-forties who, all of a sudden,

became transformed into the philosophical giant we are familiar with. There was originality in his early philosophy, in part, because Kant's attempt at bridging the growing rift between science and metaphysics went against the tide of the times. In addition, he criticized the Leibnizian-Wolffian School Philosophy and creatively revised Newtonianism instead of simply combining the two. There was also more continuity between Kant's early and late philosophies than appears at first sight. For one thing, the crisis of the pre-critical project was the catalyst that triggered the development of the critical system. Furthermore, the inception of the critical philosophy was not an abrupt conceptual breakthrough as Kant portrayed it in B xvi and B xxii of the first *Critique*, but rather a series of incremental steps that had begun with his growing disenchantment with the pre-critical project in the 1760s. Finally, the critical Kant preserved the same metaphysical assumptions he had advocated earlier (the existence of God, the possibility of moral freedom, the presence of purpose), only now, more tenuously as postulates.

In addition, and perhaps most important, the early texts are not devoid of relevance to the subsequent course of natural science and philosophy, for as often as the early Kant was dead wrong in his scientific conjectures and metaphysical constructions, he was right on the mark. It is true that the pre-critical texts concerned numerous themes which we would now regard as obsolete—such as the living forces, the ether-theory of fire, the cosmic teleology, the physical monadology, or the demonstrability of God's existence. Nonetheless, the young Kant came up with numerous insights of far-reaching consequence. On the side of metaphysics, he made contributions that would survive him. He replaced miracle-based teleologies with functionalist explanations that were intended to be compatible with scientific descriptions. He expanded the hypothesis of physical influx into a full-fledged theory of causality, thereby presenting a third option to the not altogether satisfactory alternative of occasionalism and preestablished harmony. He proposed a compatibilist resolution of the conflict between freedom and deterministic processes for the sake of permitting the possibility of moral action in this world. He recognized (already in the pre-critical period) that existence is not a property and that the traditional ontological arguments of God's existence were in need of revision. And he argued that linguistic analysis ought to be the foundation for aprioristic constructions.

On the side of natural science, Kant's conjectures foreshadowed a number of eventual discoveries, anticipating some of them by as much as two centuries. He found out that the friction caused by the oceanic tides (determined primarily by lunar gravitation) would eventually decelerate the rotation of the Earth until the terrestrial day would last as long as a lunar revolution. He suggested that the solar system accreted from a gaseous cloud that started spinning through the interplay of its own forces. He proposed that the Milky Way is a disk-shaped, dynamic system of suns that rotates around its center of gravity. He stipulated that the so-called foggy stars observable with the telescopes of the time were galaxies similar to our own. And he discovered that the rotation of the Earth affects the patterns of the monsoon

winds. Even the very essence of Kant's precritical project, the unification of natural science and metaphysics into a philosophical model of nature, remained a theme of post-Kantian thought, as the systems of Schelling, Hegel, and Schopenhauer so colorfully illustrated. Thus, despite what Kant later had to say about it, the precritical project pushed philosophy forward.

But was there such a thing as the precritical project? After all, the biggest interpretive challenge raised by Kant's early philosophy consists of its apparent incoherence. When reading through the precritical texts, it often seems as if their author jumped from topic to topic and adopted whatever perspective suited the occasion. In his influential study of Kant's life and works (1918; transl. 1981), Ernst Cassirer gave a trenchant description of this precritical oddity:

Looked at closely, [Kant's] life did not progress at all "in perfect regularity," but moved in a very irregular way toward its goals. . . . Everywhere one comes upon places at which his thought, after it is just on the point of arriving at a definite solution, suddenly steps backwards. A problem is taken up, thought through, and its solution reached—but suddenly it is shown that the conditions under which it was first worked out were not appropriate and complete enough, and hence not one step of the solution is valid, but instead the whole way in which the question is put has to be framed anew. Reticent as they normally are about questions of his inner development, Kant's letters tell us again and again of reversals of this kind. A conceptual whole is not constructed bit by bit in a steady, unbroken progression, but new threads seem continually to be spun, only to be immediately severed. (Cassirer, 1981; 92–3)

Cassirer's description has become the standard assessment of the young Kant: a second-rate thinker who moved from problem to problem, adopted different points of view, overturned them, and utterly failed to develop a coherent philosophical perspective. This interpretation naturally reinforced the dualistic picture of Kant: first an inept writer who zigzagged off into all kinds of wrong directions, only to backtrack later on—and then, a genius, who serenely built the architectonic edifice of the critical system.

The great pioneers of Kant's precritical philosophy did not challenge Cassirer's reading. As Henrich remarked in a review of Schmucker's study of Kant's precritical ethics (Schmucker, 1961), Wilhelm Wundt, Giorgio Tonelli, and Heinz Heimsoeth were the first explorers of Kant's intellectual development (Henrich, 1965, p. 254). Wundt's book on Kant's metaphysics (1924), Tonelli's monograph on the methodology and metaphysics of the precritical philosophy (1959b), and Heimsoeth's studies of various aspects of Kant's early thought (especially 1956; 2nd ed. 1971) have indeed made fundamental contributions to our understanding of the young Kant.⁴ To a greater or lesser extent, all of these scholars were wrestling with the phenomenon of Kant's precritical capriciousness. Whereas the critical period of Kant's thought exhibits a sustained perspective that would only dissolve in the *Opus Postumum*, such a sustained perspective seems absent from the

precritical period. Instead, these erratic transitions pose a problem for the interpreter of the philosophy of the young Kant.

Two recent readings may serve to illustrate this difficulty. In his book on Kant's philosophy of mind (Ameriks, 1982), which contains an informative analysis of the paralogsms, as well as valuable discussions of the precritical views, Ameriks distinguishes four precritical phases. According to Ameriks, Kant began as an empiricist, turned toward rationalism in 1756, shifted to a scepticist position in 1766, and finally adopted a quasi-critical point of view after 1768 (*ibid.*, 12–16). Beiser, in his study of Kant's intellectual development (Beiser, 1992), identifies four phases as well, but they are remarkably different from Ameriks's proposal. According to Beiser, Kant was initially infatuated with metaphysics, then became disillusioned in 1760, partially reconciled himself with metaphysics in 1766, and finally divorced himself from metaphysical concerns in 1772 (*ibid.*, 26).

Other commentators, such as Friedman (1992b), Laywine (1991; 1993), Watkins (1995a; 1995b), or Shell (1996), have followed a different course. They respond to the problem of Kant's seemingly erratic transitions by insisting on the existence of significant continuities. Friedman characterizes the young Kant as a committed and sophisticated Newtonian, who attempted to construct the philosophical underpinnings of the *Principia*. Laywine and Watkins identify Kant's concern with physical influx as the dominant theme of the early works. Shell suggests a sustained interest in the mind-body problem and in the notion of community were the unifying motifs.

I do not think that any of these mentioned investigations from the 1920s to the present have been fundamentally misguided. What interests me, is that one can tell the story of the precritical philosophy with far greater systematicity than has been attempted before. Although the precritical philosophy of nature was not a system, it did consist of preliminary investigations aimed at bringing about a system. Kant's early thought was guided by a vision of combining a modern mechanical model of physical nature with the metaphysical assumptions of a uniform structure of nature, of a purpose to the world, and of the possibility of freedom. There was coherence in Kant's early philosophy because the sustained effort of reconciling the perspectives of science and metaphysics characterized almost all of the major precritical writings up to the mid-1760s. Of course, such a reconciliation is an extraordinarily ambitious goal. The very height of Kant's ambition explains the erratic transitions noted by Cassirer. Some of these reversals of opinion occurred because Kant wrestled with his goal: sometimes he failed and had to start anew. Others, however, were not reversals of opinion at all; they were transitions, but not erratic—they were shifts of interest dictated by the sequence of issues he encountered while advancing the precritical project.

The sequence of the issues Kant investigated also sheds light on the proposals of breaking the early philosophy into distinct phases. In the 1740s, before the precritical project was underway, Kant investigated a problem in mechanics. In the 1750s, when the precritical project took off, he tried to

reconcile the perspectives of science and metaphysics over celestial mechanics and cosmology. These early interests, first in classical mechanics, then in astrophysics, led to an empiricist style of investigation that Ameriks identifies as the fundamental feature of the first period. Kant's subsequent tasks were the reconciliation of the two perspectives over the physical microstructure (by means of a monadology) and over God (by means of a rational theology). This shift in interest suggested a more rationalistic type of investigation to Kant, which Ameriks takes as the characteristic trait of the second period. As unresolved difficulties continued to accumulate, Kant's confidence in the project waned during the time that Ameriks views as the third or "sceptical period." Finally, when Kant, after the collapse of his project, began to investigate what had gone wrong, his second-order research about the failure of his previous efforts paved the way toward the first *Critique*. This is the fourth and quasi-critical period that Ameriks suggests. Although I have some reservations about Beiser's specific organization of the precritical phases, the rise and fall of the precritical period does indeed reflect Kant's changing attitude toward metaphysics: an initial infatuation followed by a laborious and increasingly strained relationship ending in divorce.

As Friedman successfully demonstrated, Kant's attitude toward Newtonian science, by comparison, remained constant throughout most of the precritical period. However, it was not the case that Kant was a straightforward Newtonian. Instead, Kant evolved from initial scepticism to the acceptance of Newton's physics to the construction of a cosmology. His cosmology was more inspired by than actually based on the *Principia*. It involved the expansion of the applicability of Newtonian physics beyond Newton's intentions and the replacement of Newton's metaphysical underpinnings with Kant's own. These metaphysical assumptions (of nature's purpose, God's existence, and man's freedom) are quite general in content. They are not the exclusive feature of a specific school of thought. Kant's arguments draw from the conceptual groundwork prepared by the Leibnizian-Wolffians and by the Pietists, but they were essentially Kant's creations. Instead of being variations of themes familiar to the School Philosophers and their religious rivals, the precritical articulation of the metaphysical underpinnings shows an independent and creative thinker at work. This broad endeavor, of fusing Newtonian science with Kantian metaphysics, was the essence of the precritical project. It is the most compelling argument for the continuity and coherence of the philosophy of the young Kant. Although the theme of the physical influx plays a significant role in the early works, as Laywine and Watkins observe, it is, in comparison to the enormity of the precritical project, a marginal issue. Thus, the theme running through Kant's early philosophy was not merely the gossamer strand of physical influx. Nor was it the question of community and the mind-body problem, as Shell tried to suggest. Moreover, it was not a bundle of related but ultimately loose strands, as Polonoff (1971) contends.⁵ The theme running through Kant's early philosophy was the thick cable of his precritical project, the reconciliation of natural science and metaphysics.

How could the young Kant intend a reconciliation of natural science and metaphysics if their actual separation was a later development? The separation came about in the last third of the eighteenth century; that is, years after the end of the precritical period. Ironically, the old Kant contributed to it. In the preface of the second edition of the *Critique of Pure Reason* (1787), he distinguished the treatment of cognitions following the “secure path of a science” from ill-fated attempts that are “a mere groping about” (B vii).⁶ Natural science is a success story. Its revolutionary strategy of testing hypotheses through deliberate experimentation has put natural science on the “secure path of a science.” In natural science, reason approaches nature “in the capacity of an appointed judge who compels the witnesses to answer the questions that he puts to them” (B xiii). But metaphysics has been an all-out failure. It is, as Kant remarked, “a combat arena . . . in which not one fighter has ever been able to gain even the smallest territory and to base upon his victory a lasting possession” (B xv). In contrast to the procedure of natural science, the “procedure of metaphysics has thus far been a mere groping about, and—worst of all—a groping about among mere concepts” (B xv). The divorce of natural science and metaphysics was the final outcome of a problematic association that had been steadily deteriorating throughout the eighteenth century. Kant had striven for their reconciliation in the decades before the *Critique*. The precritical project had emerged in the phase of estrangement of the two approaches to nature, and it had been a last-ditch effort of saving the difficult marriage of natural science and metaphysics before it was too late.

That the precritical project was an attempt to save the worsening relationship of natural science and metaphysics is also what makes Kant’s early philosophy of nature unique. Kant recognized the rift between the scientific and metaphysical perspectives, and he wanted to do something about it. In his view, a reconciliation was needed because both perspectives are indispensable. Natural science supplies us with knowledge of the physical world; metaphysics provides us with answers to our questions about the intelligible framework of the physical world. This was Kant’s precritical position. But most philosophers in eighteenth-century Germany and elsewhere reacted differently to the impending split between the two approaches. The School Philosophers, for instance, did not take the threat seriously. Their systems evaded the conflict by compartmentalizing the two perspectives. The differences between the two perspectives mattered little to Wolff and his disciples because, in their view, the two approaches concerned distinct components of reality. Natural science was supposed to be about the empirically accessible and quantifiable surface layer of nature, and metaphysics concerned the rationally securable and nonquantifiable essence of the world. Those, on the other hand, who did take the threat seriously did not care for a reconciliation, because they tended to be partisan to one of the perspectives, hoping that it would triumph over the other. The Pietists fought against the emerging new science by challenging its legitimacy on metaphysical and theological grounds, whereas the French *philosophes*, the British empiricists,

and the Newtonians intended to discredit the established metaphysical systems with the weapons of Ockham's razor, methodological rigor, and common sense.

Johann Christoph Gottsched's *Erste Gründe der Gesamten Weltweisheit, darinn alle Philosophische Wissenschaften in ihrer natürlichen Verknüpfung abgehandelt werden* (1733/4; "The First Grounds of Complete Philosophy, in Which All Philosophical Sciences Are Treated in Their Natural Connection") is characteristic of the complacency of the School Philosophers. In the first volume of his work, on theoretical philosophy, Gottsched repeatedly refers to Newton's work. He paraphrases the laws of motion, mentions the determination of the speed of light, describes the use of the prism for the spectral analysis of sunlight, and explains the celestial mechanics of the solar system. Over many chapters, Gottsched supplies the reader with a nicely organized account of the scientific knowledge of the day. In the same volume, he also expounds the basic principles of ontology, describes the nature of substances, discusses the cosmological structure and the physicotheological perfection of the world, explains the mind-body interaction, proves God's existence, and identifies God's properties. Gottsched's "Complete Philosophy" is an idyllic garden of Eden where the lion of science lays down next to the lamb of metaphysics. Newtonian physics and rational speculation live in a peaceful and mutually profitable coalition. Physics, which Gottsched defines as the science of the material bodies (#602), generates knowledge about the properties of bodies by empirical means (#666). But as the senses have access only to the exterior of things, the work of the physicist is supplemented by the work of the metaphysician who illuminates their interior with conceptual devices (#398; ##664–666).

Among those who acknowledged the presence of tensions and took sides, the defenders of metaphysics were typically German. Moses Mendelssohn championed the superiority of metaphysical cognition in his *Abhandlung über die Evidenz in Metaphysischen Wissenschaften* (1764; "Treatise on the Evidence in the Metaphysical Sciences"). Crusius argued in his *Physics* that the use of mathematics in philosophical investigations of nature is misguided. Because he believed to have demonstrated that mathematical terms have nothing to do with real entities, he supposed that the quantitative approach used by Newton and his ilk was a dead end. On the other side of the fence was a growing alliance of European thinkers who recognized the merits of Newton and the faults of the metaphysicians. In the *Lettres à une Princesse d'Allemagne sur quelques sujets de Physique et de Philosophie* (3 vols., 1768–72; translated as "Letters on Different Subjects in Natural Philosophy," 1833), Leonard Euler took a swipe at the School Philosophers and ended his "Letter on the Systems of the Monads of Wolff" (1760) with the withering remark that "these gentlemen have no knowledge of the real nature of bodies." Voltaire played the same tune. He published an introduction to Newtonian physics, the *Éléments de la philosophie de Newton* (1738; "Elements of Newton's Philosophy") while ridiculing the pretenses of the metaphysicians in *Micromégas* (1752), *Candide ou l'Optimisme* (1759), and other works. David

Hume, the sober Scot, concluded his *Enquiry Concerning Human Understanding* (1748) with the famous polemic,

If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, Does it contain any abstract reasoning concerning quantity or number? No. Does it contain any experimental reasoning concerning matter of fact and existence? No. Commit it then to the flames: for it can contain nothing but sophistry and illusion.

Thus, the young Kant was pretty much alone in his venture. The thinker who exerted the greatest influence on him was his teacher Martin Knutzen. Only ten years older than Kant, he taught philosophy, physics, and astronomy at Königsberg, where he had been promoted to the post of an associate professor (*ausserordentlicher Professor*) at the tender age of twenty-one. In his well-received *Philosophischer Beweis von der Wahrheit der christlichen Religion* (1740; “Philosophical Demonstration of the Truth of the Christian Religion”), Knutzen couched Pietist tenets in a school-philosophical framework in order to argue against the heresy of the “freethinkers,” the English Deists. He wrote a logic textbook, the *Elementa philosophiae rationalis seu logicae cum generalis tum specialioris mathematica methodo demonstrata* (1747; “A Mathematical Exposition of the Elements of Basic and Advanced Rational Philosophy or Logic”), as well as a sophisticated treatise on the mind-body problem, the *Commentatio philosophica de commercio mentis et corporis per influxum physicum explicando* (1735; “Philosophical Commentary on the Interaction of Mind and Body, Explained by the Physical Influx”), which exerted a considerable influence on Kant. He also published some papers on mathematical problems and penned a larger, ill-fated work in astronomy, the *Vernünfftige Gedancken von den Cometen* (1744; “Rational Thoughts on Comets”). Knutzen taught Kant metaphysics and natural science. He introduced Kant to Newton’s works, and he embodied an unlikely synthesis of Wolffian and Pietist ideas. Like Kant, he acknowledged the equal relevance of metaphysical and scientific ideas. But Knutzen’s influence on the precritical project was limited. He never addressed the question of the tension between the scientific and the metaphysical perspectives. He did not concern himself with the issue of their synthesis, and when Kant began to grapple with it in earnest, Knutzen had already passed away.

Kant’s mentor died in 1751, three years before the precritical project unfolded. But Kant still had the chance of meeting a kindred spirit. This was the great philosopher and mathematician Johann Heinrich Lambert. Here, finally, was a thinker who appreciated both the scientific and metaphysical perspectives, who worried about their tensions, and who was searching for a truce. In his *Cosmologische Briefe über die Einrichtung des Weltbaues* (1761; “Cosmological Letters on the Establishment of the Universe”), Lambert worked on the same topic as Kant had in his earlier *Universal Natural History*. The tasks and results of both works resemble each other. Both were proposals of integrating Newtonian physics into a larger cosmological frame-

work, and both contained a theory of the dynamic constitution of the universe. Moreover, in contemplating the possibility of mediating between natural science and metaphysics, Kant and Lambert realized that the biggest obstacle was the absence of a coherent methodology that would do justice to either approach. Kant struggled with this challenge in the *Prize Essay* (1764); Lambert wrote two essays on the subject at the same time, which he subsequently incorporated into his *Neues Organon* (1764; “New Organon”). Both thinkers recognized that they were—almost—soul mates. Lambert wrote to Kant about “the similarity of our ways of thinking,” and Kant replied that he had already noticed the “fortunate agreement of our methods.”⁷

But Kant’s encounter with Lambert occurred too late. He learned about Lambert’s plans when he was on the verge of giving up. The *Prize Essay* was not only Kant’s last attempt at coming to terms with the difficulties that the precritical project posed, but it also remained his only effort of discussing the methodological differences of the two perspectives. Kant and Lambert agreed that a methodology capable of unifying science and metaphysics would require a conceptual overhaul of the latter perspective—an overhaul consisting of a systematic clarification of all basic philosophical notions and their implications. Lambert became engrossed in this task. In the two volumes of the *Neues Organon*, he endeavored to isolate the simple concepts, to derive a set of logically true statements from them, and to show that some of these propositions entailed metaphysical truths about the properties of existing things. Kant, however, lost heart and admitted defeat in the *Dreams of a Spirit-Seer* (1766). When Lambert invited him to join him in his venture, Kant had already been paralyzed by doubts. The last major precritical work, the *Inaugural Dissertation* (1770), was the starting point for a new philosophical path. Having jettisoned the idea of combining metaphysics and natural science on the same level, Kant sliced his model of reality into two halves, an intelligible dimension relevant for metaphysics and a sensible dimension accessible to science.

Kant’s philosophical development is a story full of high hopes and great drama. In the precritical period, he revealed little of the conservative, circumspect caution so typical of his later work. Sometimes crudely, but always in bright colors, he painted with bold strokes a picture of reality that is enchanting in its beauty and tantalizing in its promise. Ultimately, the promise remained unfulfilled, and Kant’s fall from hope was hard indeed. But throughout the meanderings of Kant’s early journey, one notices an extraordinarily creative mind at work, a mind with an uncanny knack for combining ideas. Already in the precritical period philosophy was pushed to levels never attained before.

The Vis Viva Debate

Kant's Starting Point

*1.1 The Problem of Living Forces*

Kant's philosophy is divided into the precritical period from the 1740s to 1770 and the critical period from the 1780s to the late 1790s.¹ The gap between the two periods, from 1771 to 1780, is known as the so-called silent decade, in which Kant published little.² The unifying moment of Kant's early philosophizing was the precritical project. It involved, on one level, the reconciliation of the perspectives of natural science and metaphysics, and, on another level, the construction of a unified, nondualistic model of nature. With the precritical project, Kant hoped to establish a model of nature capable of harmonizing Newtonian physics with the main assumptions of metaphysics—the presence of purpose, the possibility of freedom, and the existence of God. Although the construction of this model of nature took Kant only a few years in the mid-1750s, the precritical period as a whole stood under its shadow. The years before 1755 prepared the ground for the envisioned philosophy of nature. They did so in a negative sense through the failure of his first work, and in a positive sense through his later Newtonian conversion. The years after 1756 saw the completion of the project and Kant's subsequent doubts. In the early 1760s, he established a bridge from nature to God and formulated the methodology of his scientific-metaphysical model of nature. But in the mid-1760s, the struggle with the

second-order question of method had increasingly destructive aftereffects, shaking the edifice until it collapsed. This failure left a void in which Kant would eventually erect his critical system. The fate of the young Kant is the story of an ambitious philosopher driven by the hope of solving the big questions of metaphysics with big answers, but who was forced to realize that his answers had failed and that the big questions loomed larger than ever.

The precritical philosophy begins with the *Thoughts on the True Estimation of Living Forces* (wr. 1746–7, p. 1749), but this is not the starting point for the precritical project. Kant pursued the precritical project after the *Living Forces*. When writing his first book, Kant had not yet experienced the Newtonian conversion that would dominate his thoughts on nature for the rest of his life, and he had not yet given thought to the grand issues of metaphysics that would govern the precritical project. In this regard, the *Living Forces* occupied a position all on its own. However, the precritical project of the 1750's did not just emerge after Kant's first book; it was that book's indirect consequence. The strategy of the *Living Forces* foreshadowed the strategy of the later and grander endeavor. In the *Living Forces*, Kant intended to remove the confusions surrounding the phenomenon of force by arguing that one needs to give both quantitative-physical and qualitative-metaphysical accounts their due place. He would eventually expand this idea into his overall position; the resolution of any general phenomenon of nature, he came to believe, required such a two-prong approach. It was Kant's firm conviction that both the quantitative investigations of physics and the qualitative explorations of metaphysics have their relevance. To the extent they are at odds with each other, they define the task of the natural philosopher as the determination of the manner of their consistent copresence.

Kant wrote the *Thoughts on the True Estimation of Living Forces* as a student at the Herzog Albrecht University (the "Albertina") of Königsberg before financial problems following the death of his father forced him to drop out of school and earn his living as a tutor. The book was an exercise in natural philosophy designed to identify the metaphysical nature and mathematical formula of physical force. Some commentators (de Vleeschauwer, 1939; Beiser, 1992) regard the *Living Forces* as Kant's dissertation, but in fact Kant did not earn any degree with the book (it would take him ten more years to complete a master's thesis and doctoral dissertation).³ Kant composed most of the manuscript in 1746 and submitted it to the censor. The dean approved it, and a wealthy relative helped Kant to finance the printing. The publisher Hartung slated its publication for 1747 but delayed the actual press until 1749. Kant's philosophical debut was a false start. He later considered the *True Estimation of Living Forces* a thorough embarrassment, which, for all practical purposes, it was. Not only was Kant incapable of resolving the problem of force, but also unbeknownst to him, Jean Le Rond d'Alembert had already published a theory that effectively settled the debate three years before Kant turned his mind to it. In the *Traité de Dynamique* (1743), d'Alembert argued for a scientifically promising conception of force and an assessment of the relevant formulas that implied the correct mathematical

resolution of the controversy. In 1758, d'Alembert added a "Discours Préliminaire" to the second edition of the *Traité*, explicating the philosophical aspects of his resolution.

But what always distinguished Kant was his extraordinary intellectual honesty. He admitted to himself the extent of his initial failure and carefully learned the painful lesson it entailed. When he realized that the Cartesian mechanics was an insufficient physical account of force, he abandoned it and opted for Newton's physics. When he recognized that the Leibnizian-Wolffian metaphysics was ailing from problems, he broke with the School Philosophy and constructed his metaphysical demonstrations on a more sophisticated and independent basis. The *Living Forces* cast a long shadow over Kant's subsequent endeavors. The deficiencies of the philosophy of mathematics that both inspired and impeded the theory of force led to Kant's attempt, in the *Universal Natural History* (1755), to come to terms with the relation of quantitative and qualitative approaches to physical phenomena. The confused reflections over the relations of substance, interaction, and world, in the first part of the *Living Forces* (#4–14), prompted Kant, in the *New Elucidation* (1755), to systematically clarify these notions and formulate a general ontology of nature. The vague assumption that matter involves an entelechy, which stands at the beginning of the *Living Forces* (#1, I 17), required an explanation, and Kant eventually unpacked it in terms of a general dynamic theory of matter in the *Physical Monadology* (1756). The precritical project grew out of the systematic reappraisal of the suppositions of the *Living Forces* and constituted the philosophical lesson learnt from the errors of the initial theory. In a similar manner, Kant would later learn an epistemological lesson from the metaphysical shortcomings of the precritical project, drawing, with the critical turn, second-order conclusions from the project's first-order difficulties.

The *Living Forces* was a contribution to a protracted controversy that concerned two questions. First, is there such a thing in nature as a "living force," a self-generating, essential *vis viva*, distinct from sheer mechanical pushes and pulls? Second, can the proposed measurement of the living force (as the product of mass and the square of velocity) be justified in the face of the measurement of sheer mechanical or "dead" force, the *vis mortua* (as the product of mass and velocity)? The controversy over the conception of force had started in 1686 between Leibniz and the Cartesians and had engaged many of the leading philosophers of nature of the last two generations. Since the debate had gone on for almost sixty years before Kant joined the fray, there was a plethora of literature on the subject, and the continuous exchange of proposals and counterproposals had given a highly technical character to the issue. Kant, who intended to settle the controversy once and for all, wrote the *Living Forces* as a technical response to both of the adversarial views—the Leibnizian position in favor of *vis viva*, and the Cartesian (and, to a lesser extent, the Newtonian) position repudiating it.

Accordingly, an unsuspecting reader of the *Living Forces*, who thinks that the B-Deduction in the *Critique of Pure Reason* is the most difficult text in Kant's oeuvre, will be in for a surprise. Studying Kant's first book is an

extraordinarily strenuous and frustrating undertaking. Since the treatise is about a topic that is now in the dustbin of history, the reader will encounter theoretical constructs that have nothing to do with our understanding of force, as well as deceptively familiar notions that sound like our modern concepts while actually involving the metaphysical underpinnings of the *vis viva* debate. The *Living Forces* was not written for a general audience, but was addressed to the participants of this debate. Its language mirrors the terminology of the natural philosophers in the seventeenth and eighteenth century and bears little resemblance to our modern scientific vocabulary. Reading Kant's contribution to the *vis viva* controversy is a disorienting experience. It is similar to the dizziness felt when entering a room full of specialists discussing an unfamiliar issue in an unfamiliar jargon.

The difficulties of reading the *Living Forces* are compounded by the fact that its twenty-two-year-old author was an inexperienced writer. Sentences are always verbose and complicated, frequently clumsy and ambiguous, and sometimes grammatically incorrect; positions are seemingly defended only to be repudiated later on; the solution to the problem of force is different in the beginning and at the end of the treatise; the structure is rambling and consists for the most part of an excruciating enumeration of flawed objections to flawed arguments. It is not exaggerated to say that the *Living Forces* is the worst text Kant ever wrote. But since his first book is the key to things that would come later, the effort of penetrating its forbidding terrain has its rewards. We cannot ignore the *Living Forces* if we want to understand Kant's philosophical development. The apparent difficulties of the treatise will dissolve if we begin our account with a look at the debate which the *Living Forces* was intended to settle. We need to recapitulate what happened in the smoky room full of quarreling specialists before we, and Kant, appeared on the scene.

The problem of living force arose in the context of seventeenth-century mechanics. The debates over force involved a whole range of issues about the nature, manifestations, measure, kinds, and conservation of force. In particular, the controversy over living force concerned two main questions. How is force measured? Is force only a quantity? Measuring force requires the identification of the right formula and thus involves a quantitative approach, while examining force as a quality is on a different level, requiring a philosophical clarification of the fundamental nature of force. That these two issues were not clearly distinguished in the controversy led to terminological confusions and misunderstandings among the participants, making the problem of living force almost impossible to solve. Even if a certain formula applied to some experimental contexts and measured something, it was not clear what it was that had been measured.

The debate over the two questions occurred between two groups, the one siding with Descartes, the other following Leibniz. Descartes had argued that force is a quantity and nothing more. Force should be measured as the product of velocity and the quantity of matter. Because force, in Descartes's measurement, is thus directly proportional to velocity, it is directly propor-

tional to motion. The Cartesians conceived of force accordingly as the *quantity of motion*. Leibniz rejected this description. He thought that force was neither the particular quantity of motion nor reducible to a quantity in general. Leibniz considered force a quantity as well, but in contrast to Descartes, he was not persuaded that it was *only* a quantity. He viewed force as an essential and qualitative component of matter. According to Leibniz, this dynamic essence of matter or the substantial force has kinematic effects that we can measure. So force, in this view, is a quality that involves a quantitative aspect. Leibniz claimed that this quantitative aspect should be measured as the product of the *square* of velocities and the quantity of matter. Leibniz's label for this conception of force was "living force" (*vis viva*). Depending on the context, "living force" referred to the essential quality as such, or to the formula describing its quantitative manifestation. In the jargon of the time, "living force" denotes the Leibnizian force and its measurement as the product of the quantity of matter and the square of velocities, whereas "dead force" or "dead pressure" (*vis mortua*) denotes the Cartesian force measured as the product of the quantity of matter and velocity.

The term "quantity of matter," which occurs in both conceptions of force, has disappeared from the vocabulary of modern physics. In his discussion of moving bodies in *Le Monde* (1633; see *AT* 11:51), Descartes identified "quantity of matter" as *size*. In a subsequent letter, to Christiaan Huygens on 5 October 1637, he identified "quantity of matter" with *weight* (cf. *AT* 1:435–9). The context of this latter identification is the mechanics of simple machines. In the *Specimen Dynamicum I* (1695), Leibniz referred to the "quantity of matter" as "magnitudo corporis" (magnitude of a body), which has been variously rendered as *magnitude* (Loemker, 1956, 2:725), *simple mass* (Loemker, 1956, 2:726), and *size* (Ariew and Garber, 1989, 128). In fact, the degree of a body's resistance to changes in place and motion depends on the body's *mass*—a concept that nobody understood before Isaac Newton's definitions of mass and inertia became public knowledge.⁴

We can trace the origin of the concept of mass to the drafts of *De motu corporum in gyrum* (1684), a short treatise that Newton wrote and sent to Edmond Halley in response to the latter's questions about celestial mechanics. Hypothesis 2 of *De motu* states, "Every body by its innate force alone proceeds uniformly to infinity in a straight line, unless it be impeded by something extrinsic" (*Add.* 3965.7; *fol.* 55r; see De Gandt, 1995, 18–19). This was the germ of the first law of motion. In the *Principia*, Newton generalized the hypothesis to include rest, and he dropped the stipulated relation of inertial motion and innate force. The thus revised law of inertia of the *Principia* states, "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it" (*M* 1:13). As Newton explains, "body," "mass," and "quantity of matter" mean the same thing in his terminology; they each refer to the measure of matter that arises from its density and bulk conjointly (definition 1, see *M* 1:1). Hence, in Newtonian physics, mass or quantity of matter is the measure of the amount of matter in a body which determines

the body's resistance to acceleration by an applied force. Newton considered mass to be constant. (We know through Einstein that mass varies with the velocity of a body and that it represents a stockpile of energy.) Although Newton had published his findings in the late seventeenth century—the first edition of the *Principia* appeared in 1687—it would still take decades before Newton's definition of the quantity of matter in terms of mass was generally accepted.

If we (anachronistically) substitute “mass” for the ambiguous “quantity of matter,” then both Descartes's quantity of motion (mv) and Leibniz's living force (mv^2) point to actual physical quantities. The Cartesian product of the quantity of matter and speed anticipated the product of a body's mass and its linear velocity that we now call *momentum*. The Leibnizian product of the quantity of matter and the square of velocities is the precursor to *work*. The quantity that we now label *force* derives from Newton's second law of motion (the law of acceleration) and Euler's quantitative formulation of this law. Thus, force is actually the product of mass and acceleration. Neither Descartes nor Leibniz succeeded at identifying force, but both Cartesian momentum and Leibnizian work are related to it. Momentum (in modern notation, $p = mv$) is an aspect of force in that force ($F = ma$) denotes the rate of change of momentum. Work or kinetic energy ($k = \frac{1}{2}mv^2$) is the transfer of energy to a body by an application of force. In the context of simple machines, work signifies the product of an applied force and the resulting displacement.

Both Descartes and Leibniz were influenced to some extent by Galileo, the founder of modern mechanics. In the old Aristotelian metaphysics, which Galileo overcame, force had been an entelechy and motion had been a process. For Aristotle, there was no such thing as free motion; if the cause of motion stopped acting on the object in motion, the motion of the object would cease. Galileo was the first to realize that motion is not a process requiring a continuous cause, since it persists until something external acts on it. The insight that motion is not a process made modern mechanics possible. It implied that motion must be a *state* and is thus just like rest. Although Descartes and Leibniz had different views on Galilean mechanics (which contributed to the prolongation of their dispute), Galileo's idea of considering motion to be a state served as the common ground of their rivaling theories of force.

Galileo's mechanics was an intermediate point between the old and the new. Two deficiencies prevented Galileo from constructing a full account of inertia. His mechanics lacked a concept of mass, operating with weight instead, which precluded the determination of the acceleration by a force on a horizontally moving body. It also involved a strange concept of curvilinear inertia according to which not only uniform linear but also uniform circular motions are natural states that maintain themselves without the active exertion of a force. (The idea is that a body is at rest on a horizontal plane that is actually a segment of the spherical surface of the Earth; hence, bodies are at rest on a plane whose points are equidistant from the Earth's center.)

Nevertheless, Galileo's conceptual revolution, his contention that motion and state are equivalent states, was a first step toward the notion of inertia expressed in Newton's first law of motion.⁵

In addition to his new conception of motion, Galileo's call for a new methodology shaped the *vis viva* debate and particularly impressed Descartes's followers.⁶ Nature stands continually open to our gaze, Galileo declared in *Il Saggiatore* (1623), hence, nothing in nature is intrinsically occult. Since the book of nature is written in the language of mathematics, Galileo continued, we must investigate phenomena in quantitative rather than in qualitative terms.⁷ Moreover, as Galileo remarked in the earlier *Macchie solari* (1613), we should investigate the phenomena and their properties instead of trying to penetrate the essences of natural substances.⁸ Predictably, an occult, qualitative Aristotelian entelechy that is neither empirically accessible nor suited to mathematical description cannot be part of the book of nature.

In some regards, Descartes's mechanical philosophy looks like an expansion of Galileo's methodological proposal. Three cornerstones of the Cartesian philosophy mutually reinforce the conception of force as a quantity of motion. First, since mathematics can describe physical nature, force can be described by mathematics as well, hence, force is quantifiable.⁹ Second, because matter consists only of extension, matter is wholly inert, lacks any inherent tendency to move, and does not possess any resistance to being set in motion.¹⁰ Accordingly, an active force cannot be an internal component of passive matter. Force is not only quantifiable, it is a quantity as such—there is no qualitative dynamic residue pertaining to the internal constitution of matter. Third, God is the sole causal agency of nature, and hence, essential forces cannot exist, for if they did, they would be causal agencies in their own right. Generally, then, force is merely a quantity, and since the only true dynamic source of nature is God, force is reduced to an humble *kinematic* quantity. It is simply the quantity of motion. Occult and unobservable forces such as sympathies and antipathies, congruences and incongruences, or attractions and repulsions had no place in the Cartesian world of inert bodies. Descartes freed mechanics from the quagmire of qualitative medieval conceptions, but perhaps he streamlined mechanics too much. For him, forces were relevant only in terms of external actions that are observable and measurable. Descartes equated "force" with an "action" exemplified in motions.¹¹ In this way of looking at nature, a mechanics of force could only take a kinematic form—any kind of dynamics would smack of an illicit metaphysics.

The Cartesian equation of mechanics with kinematics has interesting philosophical consequences. In a section of the *Principia Philosophiae* (1644), Descartes argued that God not only created nature and initiated its motions, but also preserves the original amount of motion in the universe. Motion is a modification of matter. Although the motion of individual parts of matter is not fixed, and local motions can increase or decrease, a fixed, unchanging quantity of motion remains in the universe. If one particle moves with twice the velocity as another particle and this second particle is twice as big as

the first, there is as much (quantity of) motion in the smaller as in the bigger, and whenever the motion of one particle decreases, the motion of another particle increases proportionally. Despite the changes we see in the world, Descartes concluded in this passage, God conserves an equal quantity of motion in matter.¹²

This particular chapter in Descartes's *Principia Philosophiae*—section 36 in part 2 of the work—was the spark that ignited the fight over living forces. What remained constant for Descartes, the sum-total of mechanical energy or force in the world, is the quantity of motion. Force is measurable such that a unit of force corresponds to a unit of motion. If two particles of unequal size move at different speeds, their quantities of motion are identical if the products of their size and speed are the same.¹³ Descartes's argument in the *Principia Philosophiae* illustrates that the subsequent *vis viva* debate involved, from the very beginning, a mixture of distinct issues. The principle of conservation was such a question, mixed into the general cluster of problems about the nature and measurement of force. Both sides agreed that there is something that is conserved in physical nature; the question was, what is it? Descartes thought that God preserves the quantity of motion; the principle of conservation accordingly concerns that quantity which is expressible in the formula mv . Leibniz, on the other hand, thought that God preserves *vis viva*; for him, the principle of conservation is about the quantity of force describable by the formula mv^2 .

It seemed to Descartes that experimental data supported his measurement of what he called force. The same formula applies to the mechanics of simple machines such as levers, pulleys, and balances. For Descartes, force is always the product of two simple factors. He wrote Huygens (in the aforementioned letter from 5 October 1637; *AT* 1:435–6) that all simple machines are based on the principle that the same force can raise a body weighing 100 pounds 2 feet, another body of 200 pounds 1 foot, and a body of 400 pounds $\frac{1}{2}$ foot. In each case, an equal force is involved, which means that force must be calculated as the weight of a body multiplied by its vertical displacement. To use the Cartesian mv formula as a universal measurement of force in all kinematic situations presupposes that v can stand for velocity in the case of bodies in real motion, as well as for displacement in the case of the virtual motion involved in simple machines.¹⁴ Leibniz agreed with Descartes that we can express force mathematically, but he did not believe this would capture the nature of force. Descartes viewed the extension of matter, which is inert, as the final component of physical reality. By assumption, the structure of material reality is inert; by definition, force is not inert. It follows, Descartes concluded, that force cannot be part of the structure of material reality. Instead, force is reducible. The Cartesian force can be reduced to a number, and what appears as a *dynamis* is in fact the kinematic quantity of motion. In the Cartesian picture, motion was more real than force.¹⁵

In the Leibnizian picture, motion and force trade their ontological places. In Leibniz's philosophy, force becomes a genuine entity, and motion is just a relation among phenomena. As the famous definition of substance as *un être capable d'action* testifies, there is an irreducible dynamic component to

reality.¹⁶ This irreducibility does not preclude the mathematical expression of force, but it precludes the mathematical reduction of force. Force can be quantified as a mechanical magnitude, but force is not a quantity as such. As Leibniz argues, “everything happens mechanically in nature, but . . . the principles of mechanism are metaphysical.”¹⁷ Force is such a metaphysical principle of mechanism; it subsists as an ultimate quality and is the dynamic essence of matter.

The *vis viva* controversy got under way with Leibniz’s paper *Brevis demonstratio erroribus memorabilis Cartesii et Aliorum* (1686). The “memorable errors of Descartes and others” consisted of the assumption that the quantity of motion is conserved and of the identification of force with the quantity of motion—what Leibniz regarded as “the most famous proposition of the Cartesians.”¹⁸ According to Leibniz, this identification is erroneous because it involved two specific mistakes. First, Descartes considered v , velocity, as a positive quantity, not as a vector quantity. (Several years earlier, John Wallis, Christopher Wren, and Christian Huygens had shown that the quantity conserved in one-dimensional collision was not mv with v as a positive quantity, but mv with v as a vector quantity whose direction is variable and must be taken into consideration.)¹⁹ Next, Leibniz accused Descartes of confusing the force of motion with the quantity used in statics for the case of simple machines.²⁰ In general, Leibniz’s “brief demonstration” employed Galilean mechanics as a weapon against Descartes. A simple analysis of rising and falling bodies by means of the times-squared law (Galileo’s formula of free fall) reveals a difference between the quantity of motion and the force of motion. Obviously, if there is a difference, then force cannot be measured as the quantity of motion.²¹ In the ensuing debate, this argument became one of the crucial weapons of the Leibnizians against the Cartesian camp, and Leibniz repeated and restated it numerous times. Of course, the argument raises a question: If we cannot measure force as a quantity of motion, then how should we measure it? Leibniz proposed his solution in another version of the argument.²² A body A weighing 4 units moves in a horizontal plane with a velocity of 1 unit. The force that propels A could, in theory, raise A to a height of 1 unit. (According to Galileo’s times-squared law, the resulting height is calculated as the square of the velocity; $1^2 = 1$.) Now, imagine that A collides with a body B , weighing 1 unit and resting in the same plane. Suppose A could transfer the *full amount* of its force to B . B would then acquire A ’s force, and if Descartes’s mv -formula for force were true, B would now move with a velocity of 4 units ($F_A = 4 \cdot 1 = 4$; $F_B = 4 = 1 \cdot v_B$; hence $v_B = 4$). According to Descartes, the force that moves A of weight 4 with velocity 1 would move B of weight 1 with velocity 4. Is this true? No. Because, as Galileo found out, the height of a rising body equals the square of the body’s velocity. Descartes’s formula entails that A ’s force, now acquired by B , could raise B to the dizzying height of 16 units ($4^2 = 16$).

Leibniz recognized that the Cartesian calculation could not be true, for the height to which a body is raised is inversely proportional to the body’s weight (in our terms: mass). So, a force F that raises a body A of weight 4

to unit height would actually raise a body B of unit weight to a height of 4, but never to a height of 16. Descartes's formula conflicts with established principles of mechanics; it must be wrong because it implies an effect four times larger than expected, an effect more powerful than its cause. Descartes's measurement implies, in effect, the absurd possibility of a *perpetuum mobile*. A different formula is needed. Now, if one employs mv^2 instead of mv , Leibniz argued, such a conflict with the principles of mechanics does not arise. For then, a body A of a weight of 4 units and unit velocity will possess 4 units of force ($F = 4 \cdot 1^2 = 4$); transferred fully to B of unit weight, F_A will impart a velocity of 2 units to B ($v^2 = F/m = 4/1 = 4$; so $v = 2$). By means of the Leibnizian formula, F_A will raise B to the predicted height of 4 units (since $h = v^2 = 2^2 = 4$).²³

Leibniz did not yet speak of *vis viva* in the *Brevis demonstratio* (1686); he coined the term, in its French equivalent *force vive*, in the unpublished *Essai de Dynamique* (1691) and in a 1692 version of the same text. Leibniz mentioned *vis viva* publicly for the first time in the *Specimen dynamicum* (1695).²⁴ In the 1686 essay, Leibniz implicitly defended the mv^2 measure of force in terms of the square root of the distance of the fall of the bodies. Although Leibniz fought for the new measure of force, he did not discover it. Christiaan Huygens realized that Descartes's conservation of motion is valid in some but not in all frames of reference. Since mv is not conserved in every frame, Descartes's principle of the quantity of motion is not correct. Huygens, who tutored Leibniz in mathematics during the latter's Paris years (1672–76), discovered that the quantity mv^2 is conserved in each frame of reference; "the sum of the products of the size of each hard body multiplied by the square of its velocity is always the same before and after impact."²⁵

1.2 *The Controversies from 1686 to 1741*

Descartes had died in 1650, before the *vis viva* debate, and upon the publication of Leibniz's incendiary article in 1686, Descartes's followers took up the cause of the quantity of motion. The strange thing about the debate is that not much happened. The same arguments were repeated ad nauseam.²⁶ No group could convince the other. Predictably, the Germans sided with Leibniz, the French with Descartes, the British with Newton; the battle over *vis viva* acquired national overtones, which certainly did not help settle the issue. The first stage of the battle was waged between Leibniz and the Abbé de Catelan, who was joined in 1689 by Denis Papin. De Catelan replied to Leibniz's piece in the *Acta Eruditorum* with a paper in the *Nouvelles de la République des Lettres*. Leibniz replied to the "sçavant Cartesien de Paris" with several articles and letters. Unfazed, the abbot replied to the reply of the reply, which precipitated another shower of Leibniz's writings. The Leibniz-Catelan exchange remained essentially fruitless.²⁷ The subsequent controversy with Papin was more valuable. Papin conceded to Leibniz one element of his objection, that a *perpetuum mobile* is absurd. If it could be shown that the quantity of motion really implied perpetual motion, then it would follow

that the quantity of motion could not measure force. But this does not follow, Papin insisted, because Leibniz's objection erroneously presupposes that the complete quantity of *A*'s force can actually be transferred to *B*.²⁸ Papin was right about this, and Leibniz's further attempts to demonstrate a full dynamic transference did not work out. Ultimately, however, no opponent could defeat the other, and by the close of the seventeenth century, both sides were exhausted.

More than a decade later, between November 1715 and November 1716, the controversy flared up again. Princess Caroline of Wales, a good friend and student of Leibniz, had met a visiting English minister, Samuel Clarke, at the Prussian court, with whom she had argued over Newton's theological views and Leibniz's recently published *Essais de Théodicée* (1710). Caroline related the incident to Leibniz. Leibniz wrote her in November 1715, criticizing Newton's claims that space is God's organ for sensing the things in the world and that the world is a machine at risk of running down unless God winds it up like a watch.²⁹ Caroline forwarded this letter to Clarke. Clarke, "Newton's bulldog," reacted by sending an aggressively worded missive directly to Leibniz. Instead of simply defending Newton's views, Clarke launched an all-out counterattack against Leibniz. Stung, Leibniz fought back, and the correspondence (with Clarke writing in English, and Leibniz in French) was under way. The Leibniz-Clarke correspondence bloomed into a general philosophical debate that repeatedly touched upon Leibniz's claim of the conservation of *vis viva* (*G* 7:352, 370, 376, 387, 413–4). However, when Leibniz's death in 1716 cut short the exchange, matters remained undecided. Leibniz had simply reiterated his earlier views on living force, to which Clarke responded with general philosophical objections.

The second stage of the controversy occurred after Leibniz's death.³⁰ In the 1720s, Leibniz's philosophical allies took up the cause of the living forces. The leading exponents of the Leibnizian camp were Jacob Hermann, Christian Wolff, Georg Bernhard Bilfinger (also known as "Bülfinger" or "Büllfinger"), Marchese G. Poleni, W. J. s'Gravesande, Peter van Musschenbroek, and Jean and Daniel Bernoulli.

A brief digression on the Bernoullis is in order here, because it is easy to confuse the various members of this extraordinary family.³¹ The family lived in the German part of Switzerland; their ancestors had been Belgian Protestants of Dutch extraction. Not only are there six Bernoullis of historical importance, but three of them carry the same first name, and two others are known by three name variations each. The head of the family was Nicholas Bernoulli (1623–1708), a government leader in Basel. He had three sons, Jacques (Jakob, James), Nicholas, and Jean (Johann, John). The third son Jean fathered Daniel, and the second son Nicholas fathered yet another Nicholas ("the younger"). Jacques Bernoulli (1655–1705), the first born, was a professor of mathematics at Basel university and is the author of the *Ars coniectandi* (1713), one of the first works on probability theory. He formulated the *Bernoulli Theorem* for predicting probable frequencies of occurrence of repetitive events. Jacques was the most important of the Bernoullis with

regard to the development of probability theory. Nicholas, the second of the three sons, did little scholarly work, but his son Nicholas (1687–1759) applied probability theory to legal problems and prepared the posthumous edition of his uncle Jacques's *Ars coniectandi*. Jean Bernoulli (1667–1748), the youngest of the three sons of the elder Nicholas and an uncle to the younger Nicholas, was a professor of mathematics in Groningen and Basel. He collaborated extensively with his older brother Jacques. Jean wrote treatises on differential equations and on geometry, worked on the application of mathematics to mechanics, and expressed d'Alembert's principle of virtual velocities in analytic form. Jean's son Daniel (1700–1782) was perhaps the most remarkable of all the Bernoullis. He was a physician by training and worked as a professor in Basel and St. Petersburg. His contributions to mathematics involved the discussion of the so-called Petersburg Problem (a riddle in probability theory), the articulation of the Law of Errors, and the development of methods for applying calculus to probability theory. His contributions to physics culminated in the work *Hydrodynamica sive de viribus et motibus fluidorum commentarii* (1738), which laid the foundation for fluid dynamics. Daniel Bernoulli formulated what has become known as the *Bernoulli Principle* in fluid dynamics, which in turn expresses the *Bernoulli Constant*, a fixed quantity of the relation of pressure, velocity, and density of a current.

Two of the Bernoullis, Jean and his son Daniel, defended the *vis viva* measure. The list of the leading exponents of the Leibnizian camp continued with the Abbé Camus, who was not a Leibnizian but an ally of the Bernoullis, and with Leonhard Euler, who once defended *vis viva* before changing his mind. In 1726, Peter the Great founded the Russian Academy of Sciences at St. Petersburg which became the new home for the Leibnizian camp after the dissolution of the first Berlin Academy. The first volume of the *Commentarii Petropolitanae* (1728) immediately revealed the academy's journal as the major platform for Leibnizian publications on the *vis viva* question.³²

At the same time, Isaac Newton's influence grew in natural philosophy, but it did little to clarify this particular debate. Although Clarke had employed Newton's views as weapons against Leibniz, the public appraisal of Newtonian physics did not correspond to the frontlines of the dispute. In the 1720s, two leading Leibnizians, s'Gravesande and Musschenbroek, embraced Newton's physics while championing living forces. Other Newtonians, on the other hand, followed Clarke's lead, agreeing with the Cartesians on the rejection of living forces. Until 1732 (when Pierre-Louis M. de Maupertuis began to defend Newton as the first of the French philosophers), the lines in the *vis viva* dispute remained drawn along the already-mentioned national boundaries. Swiss, German, Dutch, and Italian voices defended Leibniz; British members of the Royal Society in London defended Newton; French natural philosophers of the Académie Royale des Sciences in Paris defended Descartes. Among the British opponents to living forces, Henry Pemberton, G. Eames, Colin Maclaurin, Samuel Clarke, and James Jurin participated in the controversy.³³ Among the French, the most noteworthy of

the Cartesians after the Abbé de Catelan was J. T. Desaguilliers (“Desaguilliers”) and J. J. d’Ortous de Mairan, whose position Kant would later discuss at length in the *Living Forces*.³⁴ The *dramatis personae* is really all that was important in this second stage of the dispute. An amazing number of mathematicians and philosophers of nature were sucked into the quarrel, but to no avail. Once again, the new surge of publications failed to settle the debate. By the early 1730s, the appeal of the issue waned. Although the problem had stubbornly resisted all attempts at solving it, the attention of natural philosophers turned to other matters.

The *vis viva* debate seemed to have ended in a draw. Around 1740, the issue that was originally a puzzle for mathematicians and mechanical philosophers became a topic among literati and intellectuals. The *vis viva* question became party talk in French salons, in particular in Mme la Marquise de Châtelet’s circle at Cirey, frequented by the likes of Maupertuis and Voltaire. The Marquise de Châtelet (“Chastelet,” “Chastellet”) was a correspondent of Frederick the Great (who, at the time of the correspondence, was still a prince and not yet “the Great”) and would later prepare the first French translation of Newton’s *Principia* (1756). She took up the Leibnizian cause and entered into a widely heard but inconclusive argument with the Cartesian Mairan.³⁵

Why was it so difficult to settle the issue of living forces? One reason has to do with the historical origin of dynamics. Dynamics had taken its cue from statics, and mechanical laws had been derived from the forces involved in the so-called simple machines such as the lever. Because of irreducible differences between statics and dynamics, the construction of a general mechanics on the basis of the lever led to inescapable confusions. In the context of the virtual motions of the lever, velocity and displacement are interchangeable, but in the context of real motions of freely falling bodies, they are not. Some “force” can be measured in the lever in terms of the quantity indicated by velocity or displacement, suggesting the mv formula. But in cases of free fall, displacement corresponds to the square of velocity instead (as Galileo’s formula of free fall shows), suggesting the mv^2 formula. Since static and dynamic situations were not sufficiently distinguished, two different formulas with overlapping applicability were floating around in mechanics. Moreover, the Cartesian mv formula, involving v as a positive rather than a vector quantity, was inadequate. Leibniz’s criticism in this regard was right on the mark. Although the level of the *vis viva* debate deteriorated after Leibniz’s death, at least its first stage was more than a worthless fight of momentum versus work.

In addition, the ambiguities of the lever resulted in ambiguous conceptions of force that burdened mechanics and the *vis viva* debate in particular. Robert Hooke, whose work was by no means atypical of the terminological despair of his time, employed “strength,” “quantity of strength,” “force,” “force of a moving body,” “pressure,” “power” all synonymously, and on one occasion, referred to force as “pressure, endeavour, impetus, strength, gravity, power, motion, or whatever else you will call it.”³⁶ As the mechanical

measurement of force was bogged down by the ambiguities of the lever, the philosophical conception of force was bogged down by terminological confusions.

The controversial status of Galilean mechanics augmented the difficulties of the *vis viva* issue. Galileo's assessment of motion as a state, in theory the common ground of Cartesian mechanics and Leibnizian dynamics, was in practice too limited to allow a reconciliation. Descartes's and Leibniz's particular interpretations of motion-states radically differed—Descartes claimed that motion and rest are nonarbitrary opposites; Leibniz, by contrast, argued for a relativistic notion of motion that was interchangeable with rest. With respect to the issue of living force, Galilean mechanics was not much help either because Galileo's times-squared law that Leibniz used in support of living force was unambiguously endorsed only by other Leibnizians. Descartes had rejected Galileo's kinematics of free fall because the continuous acceleration of freely falling bodies was at odds with the kinematics of Cartesian vortices. The times-squared law continued to remain controversial among Cartesians; those who accepted it were unsure of its sense. Furthermore, those who acknowledged the validity of Galileo's principle were not logically committed to buy into Leibniz's employment of the times-squared law on behalf of *vis viva*. In this regard, Leibniz's mechanical arguments suffered from a crucial flaw: the fall of heavy bodies, on which Leibniz's reasoning turns, involved contingent features of our world having nothing to do with the basic laws of physics.³⁷ Since it was just this fundamental nature of mv^2 that Leibniz hoped to establish—as the measure of the dynamic substratum of the world and as the dynamic quantity that is conserved in the world—these mechanical arguments were bound to fail.

One could even say that it was impossible to say which party was right because, in a sense, both parties *were* right considering the compound meanings of “force” and the multiple levels of the debate. Both formulas ($F = mv$ and $F = mv^2$) could be justified in some mechanical contexts, and both formulas obviously measured something—even though it was not yet clear what. Both camps could produce some evidence for “their” measurement. Already at the earliest stage of the dispute, in the exchange between Leibniz and Catelan, both parties acknowledged the veracity of the competing empirical evidence and thus the soundness of the rival formula. Catelan conceded to Leibniz that in some cases (involving unequal times) the mv^2 calculation is correct. But Catelan insisted that the mv calculation applies in the “general” and “more regular” cases of two bodies falling in equal times.³⁸ Leibniz, on the other hand, granted the existence of a *vis mortua* or dead pressure measurable by mv , which he saw exemplified in centrifugal and gravitational forces.³⁹ This “dead force” is not really force yet, Leibniz believed, because it concerns only the beginning (*nisus*) of motion, not motion itself. Leibniz argued that the very first, infinitely small instant of a body's fall or motion is measurable by the mv formula, whereas the mv^2 formula more accurately represents force after the initial moment and for the whole duration of a body's motion or fall (*Spec. Dyn.*, 14–16).

1.3. *The Traité de Dynamique (1743) and D'Alembert's Preface of 1758*

Trying to unravel the bundle of confusions concerning the measure and nature of force was hopeless, and the only promising approach involved cutting through the Gordian knot of the issue. This is what Jean Le Rond d'Alembert did, the historical winner of the *vis viva* debate. According to a frequently repeated story (de Vleeschauwer, 1962; Schultz, 1965; Höffe, 1994), d'Alembert rejected both the mv and the mv^2 measurements, and proposed $F = \frac{1}{2}mv^2$ as the definitive formula for force; everybody believed d'Alembert, and the controversy was over.⁴⁰ Another story (Iltis, 1970; Shell, 1996) has it that d'Alembert merely completed what others had prepared, and that he should have split the prize with Roger Boscovich or Leonhard Euler.⁴¹

The real course of events was different from either account. D'Alembert was not an independent outsider who terminated the quarrel by rejecting either position, nor was it the case that others before him had resolved the issue of forces as clearly as he did. The mechanics of d'Alembert's *Traité de Dynamique* (1743) was influenced by Newton; its methodological approach was indebted to Descartes (despite d'Alembert's harsh words about the Cartesians). D'Alembert did not so much propose a new conception of force in his *Traité* but rather tried, like Huygens before him, to reduce dynamics to kinematics, to a mere geometry of motion. In this sense, one could argue that the *Traité* sidestepped the problem of force rather than solving it. Nonetheless, d'Alembert's kinematic proposal amounted to a solution. As we know, the Cartesian formula anticipated momentum, and the Leibnizian formula anticipated work. Because "momentum" and "work" denote real and distinct aspects of physical interaction, Descartes and Leibniz were both right in their own way. This was d'Alembert's insight in the *Traité de Dynamique*. It is not true that d'Alembert rejected both formulas, as one story has it; instead, he pointed out that both the Cartesian and the Leibnizian measure were of equal validity.

D'Alembert's resolution of the *vis viva* dispute in the first edition of the *Traité* was technical and largely inaccessible to readers not well versed in the mathematical treatment of kinematics. Had Kant read it before writing the *Living Forces* (he did not), he might not have understood it very well. In 1758, a second, revised, and expanded edition of the *Traité de Dynamique* appeared; d'Alembert had added an introductory essay, the "Discours Préliminaire," containing a forceful rejection of *vis viva* in a metaphysical sense and a clear statement of the new quantitative conception of force. Ultimately, it was this "Discours Préliminaire" that ended the debate.

The *Traité de Dynamique* did not answer the question of force, which kept on playing a significant role in nineteenth-century physics (for instance, in energetics), but it succeeded in answering the question of *living* force in physical contexts. Since the *vis viva* debate was a multifaceted dispute, the first step of its resolution consisted in isolating the individual facets and

dealing with them separately. D'Alembert transformed an issue that had been a motley mixture of metaphysical and mechanical elements into a purely mechanical question. During the same time, and perhaps sped along by the publication of d'Alembert's work, Newtonian physics celebrated a belated triumph on the continent, first in France in the 1730s and '40s, and soon thereafter (in the 1740s and '50's) in Germany.⁴² Because Newton had emphatically rejected any inquiry into underlying causes, causal questions about the origin of force or the source of motion quickly fell out of fashion, and this, in turn, helped d'Alembert to have the final word in the controversy.

The "Discours Préliminaire" in the 1758 edition of the *Traité* begins with a proposal for a methodology in mechanics that has Cartesian overtones.⁴³ The more abstract and fundamental a field is, the more certainty it is capable of. Since d'Alembert's objective was to overhaul mechanics and transform it into an inquiry allowing of rigor and certitude, one must apply geometry, a more basic and certain science, to mechanics, a more advanced and specialized field (*Traité*, iii, iv). By generalizing mechanics in this way, its principles will acquire maximum clarity (v). This clarity, however, requires that the principles of mechanics be reduced to their smallest possible number; rigor and parsimony go hand in hand—"en un mot, d'étendre les Principes en les réduisant" is the key to progress in mechanics (v). D'Alembert recognized that a flood of principles, approaches, and concepts had turned the *vis viva* debate into a morass; accordingly, any progress about these things requires draining the irrelevant from mechanics. The "first and foremost" object of mechanics is motion (v), hence, dynamic questions should be approached kinematically. Mechanics should be oriented to mathematics, hence, mechanics should be a consistently quantitative investigation. Metaphysical principles are woefully lacking in clarity, and d'Alembert did not endeavor to refute them (iv)—metaphysics belongs to the irrelevant, and it does not have a place in this treatment of the issue.

Because mechanics is primarily about motion, force is relevant only in terms of the motion it produces (xvi). D'Alembert wanted to reduce talk about forces to a minimum. He refers to them only if constrained by the established terminology (for instance, by calling inertia with Newton the "force d'inertie"). Like Newton, d'Alembert could not care less about the causes of forces, or the "causes motrices." Forces are relevant only in terms of their effects. As inherent forces, they are simply "êtres obscurs & métaphysiques," whose consideration would only obfuscate matters (xvi). For all practical purposes, such metaphysical, inherent forces do not exist, and if they did, they would not make a difference. D'Alembert declared that only external causes can alter the uniform movement of a body (I.I, 22). He stripped the question of living force of its metaphysical aspect, reducing it entirely to a question of measurement. Whereas Leibniz's qualitative living force was measurable only in its phenomenal counterpart, d'Alembert transformed it into a pure quantity, equating the *force vive*, as he called it, with the measurement itself. That d'Alembert excised the qualitative aspect of

force from the investigation was both good and bad. On the one hand, it left one dimension of the issue of force unresolved (which eventually led to the resurrection of the Leibnizian notion of kinetic energy in nineteenth-century physics). On the other hand, the quantitative focus on force gave subsequent physicists unambiguous concepts to work with.

In d'Alembert's account, one cannot measure force by the space traversed, or by the time taken, or by an analysis of mass and velocity. Force is measurable only through the obstacles encountered and the resistance that it gives these obstacles (xviii). Force is now relevant as a quantity determined by the resistance of its obstacles. The mass-velocity formulas of Leibniz and Descartes do not capture force as such, only some of its specific aspects. If the resistance of an obstacle destroys the motion of a colliding body instantly, it will give rise to an equilibrium. The resistance of the obstacle and the impetus of the colliding body cancel each other out in an equilibrium. Both obstacle and body possess the same product of mass and virtual motion (*vitesses virtuelles*). In this case, force is measured as the sum of resistances and can be represented by the Cartesian *quantité de Mouvement* (xx). On the other hand, if the resistance of an obstacle does not destroy the motion of a colliding body instantly, but destroys it gradually (as a spring does for instance), then one deals with a case of retarded motion. But in that case, the number of obstacles overcome is like the square of the velocity of the moving body; force is then measured by the absolute quantity of resistances, and the Leibnizian *force vive* applies (xx).

D'Alembert concluded that all the difficulties of the *vis viva* issue boil down to the decision of whether the kinematic situation to be examined falls in the category of equilibrium or in the category of retarded motion (xxi). Now, this conclusion is partially false. As the classification of obstacles and the curious connection of obstacle-type to force-type show, d'Alembert was still confused about work and momentum. But what is fundamentally right about this conclusion is its underlying thought: although neither the Cartesian nor the Leibnizian formula succeed in capturing force as such, both capture specific and distinct aspects of force. D'Alembert apparently sensed that his overt and erroneous conclusion, linking force-type to obstacle-type, was wrong. He added, almost as an afterthought, that one could also apply the mv formula to cases of retarded motion—if one wants to estimate, in such cases, force *qua* the sum of the resistances of the obstacles. So, it all depends on what one wants to measure. D'Alembert had realized that both formulas are of general validity in expressing different quantifiable aspects of mechanical phenomena. Finally, the rivalry between the two formulas was exposed as an illusion.

If we want to make sense of this convoluted controversy from our contemporary vantage point, then we shall see that the *vis viva* debate resulted in a scientifically appropriate employment of the Leibnizian and Cartesian formulas. In our terms, the formula of Leibniz's *vis viva*, or $F = mv^2$, anticipated the modern concept of *work*. Work is the product of a force and a distance and depends on the angle between force and displacement. It is a

scalar quantity, a quantity involving magnitude only, like kinetic energy, to which work is closely related. Work could also be defined as the transference of energy that equals the product of the distance through which the point of application moves, and the component of force acting in the direction of the force's moving point of application. By comparison, the formula of Descartes's quantity of motion anticipated the *momentum* of a moving particle, usually expressed as $p = mv$. Momentum is related to the impulse of a moving particle. Unlike work or kinetic energy, momentum and impulse are vector quantities, quantities involving both magnitude and direction. The impulse of a force refers to the product of the force acting on a particle and the time during which it acts. This product can also be expressed as the product of mass and velocity of the particle at two different times, and this product is called "linear momentum."

The irony of the debate was that both adversaries hoped to define *force*, but both defined something else. After all, neither mv^2 (work) nor mv (momentum) are the same as force. The modern concept of force derives from Newton's second law of motion and is equal to the product of the mass of a moving particle and the particle's acceleration. Our measure of force is the legacy of Euler, who put Newton's second law into the formula $F = ma$.⁴⁴ Although Leibniz and Descartes failed to identify force as such, unintentionally arriving at work (kinetic energy) and momentum, they were still in the ballpark. Loosely speaking, Leibniz's formula correctly expresses a spatial aspect of Newtonian force, and Descartes's formula correctly expresses a temporal aspect of Newtonian force. In short, kinetic energy is the Newtonian force F acting through space; momentum is F acting over time.⁴⁵

The loser of the debate was *vis viva*, understood in its qualitative sense as a dynamic property of matter. But this exile from scientific discourse was only temporary. D'Alembert's solution rested on a deliberate choice: he wanted to investigate what could be investigated quantitatively and simply ignored the rest. This choice settled the debate, but it did not refute the possibility of a dynamic property of matter. Many followers of Newton latched on to precisely this option. According to Newton, matter was a basic building block of nature, and force was imposed on this essentially passive matter. But what was force by itself? Was it an active constituent of nature? Or was it rather a mere phenomenon reducible to motion? Already in the early *Letters to Serena* (1704), the deist John Toland reinterpreted Newton's ambiguous concept of force as an immanent property of matter. Toland argued that motion is inherent in matter—that this essential motion or "autokinesy" is tantamount to the gravitational force, and that therefore matter is active.⁴⁶ Toland's conception of an active matter turned out to be a success, and later Newtonians viewed force along similar lines, such as Robert Greene in *The Principles of the Philosophy of Expansive and Contractive Forces* (1727), or Joseph Priestley in his *Disquisitions relating to Matter and Spirit* (1777).⁴⁷ Kant, after the *Living Forces*, was no exception, arguing for forces immanent to an active matter in the *Universal Natural History* (1755) and

the *Physical Monadology* (1756), although he would now carefully avoid the terms “living force” and “vis viva.”⁴⁸

At the end of the eighteenth century, living force and *vis viva* returned from their exile. The vitalists resurrected these terms but disconnected them from their origin in mechanics. The physician Herrmann Boerhaave rehabilitated *vis viva* for chemistry; the biologist Albrecht von Haller (whose poems Kant would quote in the *Universal Natural History*) advanced a theory of irritability that suggested a living force for physiology on the basis of the contraction of muscle tissues. Later, Johann Gottfried Herder would reintroduce these terms into teleology with his treatise *Vom Erkennen und Empfinden der menschlichen Seele* (1778).

What all of these thinkers, from the Newtonians to the vitalists, suspected, was that there were some kinds of dynamic properties in matter that still awaited clarification. They were, of course, right. Through Faraday’s work on electricity and Maxwell’s on electromagnetism, the *dynamis* of matter returned in the nineteenth century to physics in the guise of energy, finally acquiring a form that permitted its systematic investigation.